

Running Control of Quadruped Robot Based on the Global State and Central Pattern

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Abstract: For a real-time quadruped robot running control, there are many important objectives to consider. In this paper, the running control architecture based on global states, which describe the cyclic target motion, and central pattern is proposed. The main goal of the controller is how the robot can have robustness to an unpredictable environment with reducing calculation burden to generate control inputs. Additional goal is construction of a single framework controller to avoid discontinuities during transition between multi-framework controllers and of a training-free controller. The global state dependent neuron network induces adaptation ability to an environment and makes the training-free controller. The central pattern based approach makes the controller have a single framework, and calculation burden is resolved by extracting dynamic equations from the control loop. In our approach, the model of the quadruped robot is designed using anatomical information of a cat, and simulated in 3D dynamic environment. The simulation results show the proposed single framework controller is robustly performed in an unpredictable sloped terrain without training.

Keywords: nonlinear oscillator, neuron network, global state.

1. Introduction

For a dynamic legged robot system, researchers suffer from lack of systematic framework to design a controller, since a locomotive system has highly nonlinear dynamics and discontinuities of the dynamics caused by the transition of support modes(e.g. single support, multiple support or ballistic state).

One of the aim in locomotive system is not telescopic type legs made up of prismatic joints, but articulated type legs made up of revolute joints [1]. Using articulated type legs, there are studies based on reference trajectories [2] [3] [4]. Arikawa et al. [4] divided motions into several states and controlled by solving inverse dynamics in each state. By solving inverse dynamics, the controller may have an accurate control performance, but one of the problem is that the controller needs a lot of calculation time. And other problems exist such as too many parameters, trajectory regenerations because of model errors [5].

A continuous motion control could be realized by switching controllers according to support modes [6]. This control framework involves controllers for each situation. But, if controllers are changed along support modes, then the system may be unstable because of the discontinuities between controllers.

Some researches treat running control problem using artificial intelligence. Krasny [7] proposed the set-based stochastic optimization, and Marhefka et al. [8] used a fuzzy theory and performed a 2D simulation study for a high speed gallop. But, these fuzzy theory based approaches need training.

Most of those earlier studies employed precise models of a robot and an environment, and involved planning joint trajectories as well as controlling joint motions on the basis of an analysis of the model. However, when a legged robot moves quickly in a variety of places, a method consisting of modeling, planning and control is neither effective nor adaptable for the reason identified by Brooks [9] in his proposal of the behavior-based approach. Moreover, even if various meth-

ods are considered for a modeling error, it is hard to know an unpredictable environment such as the inclination of the slope.

It is well known that animal's locomotion is mainly generated at the spinal cord by a combination of a CPG (Central Pattern Generator) and reflexes receiving adjustment signals from a cerebrum, cerebellum and brain stem. The motion methods of animals seem to offer a solution for the control method of legged robots. Kimura [10] proposed the CPG and reflexes based control law for walking of a quadruped robot, which dose not requires a precise model. Zhang's control law [11] is a simple and case by case for running of quadruped robot. These approaches based on biological concept have an ability to adapt the robot to an unknown environment.

In this paper, the first and second goals are how we can design the control law with robustness to an unknown environment and calculation efficiency. To achieve this goal, we introduced the global state proposed by Taga [12] [13] [14] and CP concepts, and Matsuoka's [15] [16] neuron model with its network are considered as a fundamental theory. Additional goals are a single framework controller to avoid discontinuities during transition between multi-framework controllers and training-free controller. The central pattern based approach makes the controller have a single framework, and the global state dependent neuron network induces adaptation ability to an environment. However neuron outputs are generated without a consideration of gravity effect. So, PD controller is introduced into the control architecture to maintain neutral positions of each joint. The proposed control law is simulated in 3D dynamics environment to validate the performance with the quadruped robot model designed using anatomical information of a cat.

The remainder of the paper is organized as follows. Section II develops the proposed control law, and the simulation results that validate the performance are presented in Section III. Section IV presents conclusions.

2. Proposed Control Law

In this section, the proposed control law will be developed. First, we introduce the Matsuoka's [15] [16] neuron model and its network model, then fixed neuron network will be constructed what we call the basic neuron network. And the hypothetical global state and the hypothetical diagram which are the key idea in this study will be proposed. Moreover, flexible neuron network with neuron inputs will be developed to make global states have some sequences such as a chain like relationship.

2.1. Basic Neuron Network

Matsuoka [15] [16] proposed a neuron model which consists of two ordinary differential equations, and neuron models are networked by adding connecting weights, $a_{(\cdot)}$, in neuron models as (1). In this equation, u_i is a membrane potential of the neuron body, u_0 a constant input as an impulse rate, y_i a firing rate or output of the neuron, Tr_i a time constant. f_i is the variable that represents the degree of fatigue or adaptation in the neuron, and b is the parameter that determines the steady-state firing rate for a constant input. Ta_i is the time constant that specifies the time lag of the adaptation effect.

If two neuron models are networked with a mutual inhibition, outputs of these neurons show an oscillation. So, these two neurons are called a neural oscillator unit. In Fig. 1, small circles with a number are neurons, and neural oscillators which are in charge of one revolute joint consist of two neurons. The network with the connecting weight, a_{fe} , is constructed to satisfy the property of a neural oscillator.

In this study, the target motion is a bound gait, so the network with the connecting weight, a_{cross} , is constructed to move left and right legs simultaneously as shown in Fig. 1. These two networks what we developed using connecting weights are called the basic neuron network, and shown in (2).

$$\begin{aligned} Tr_i \cdot \dot{u}_i(t) &= -u_i(t) + \sum_{j=1}^{16} a_{ij} \cdot y_j(t) + u_0 - b \cdot f_i(t) \\ Ta_i \cdot \dot{f}_i(t) &= -f_i(t) + y_i(t) \\ y_i(t) &= \max(0, u_i(t)), \quad i = 1 \sim 16 \end{aligned} \quad (1)$$

where

$$a_{ij} = a_{ji} = \begin{cases} 0 & (i = j) \\ a_{fe} & (i = 2n - 1, j = 2n, n = 1 \sim 8) \\ a_{cross} & [(i = 4l - 3, j = 4l) \\ & (i = 4m - 2, j = 4m - 1), l, m = 1 \sim 4] \end{cases} \quad (2)$$

We determined the value of parameters by Matsuoka's criteria [16] for stable oscillation and by characteristics of a running motion. Tab. 1 shows these parameters.

2.2. Proposed Global States

Taga [12] used a concept of global states which represent a sequence of a gait, and performed a simulation study for a biped walking. Now, we propose hypothetical global states for a quadruped bound gait as Fig. 2, and the hypothetical

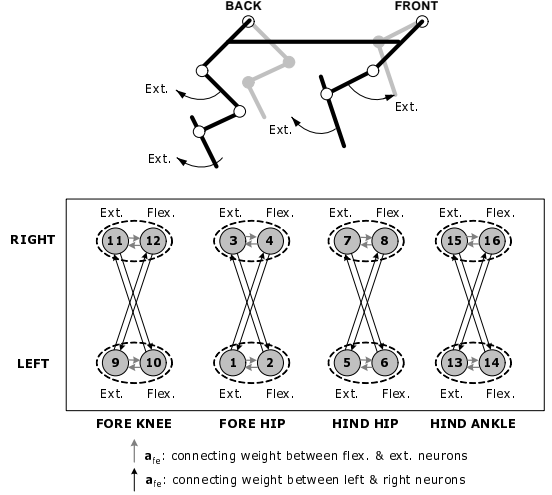


Fig. 1. Basic neuron network. There are two objectives. One is to inhibit each paired neuron in a neural oscillator. And another is to move left and right legs together.

Table 1. The parameters of the basic neuron network.

parameter	neuron $i = 1 \sim 8$	neuron $i = 9 \sim 16$
Tr	0.002	0.0035
Ta	0.0235 ($12.0482 \times Tr$)	0.0418 ($12.0482 \times Tr$)
b	2.0	2.0
u_0	6.0	6.0
a_{fe}	-2.0	-2.0
a_{cross}	-1.1(-1.0)	-1.1(-1.0)

diagram described a sequence of firing of neurons in each global state is also proposed as shown in Fig. 3. There are 4 global states divided by contact information between a foot and an environment. Furthermore, in global state 2($Sg2$) and 3($Sg3$), each global state is divided by specific angle, ψ , what we call the 'global angle'.

Neuron patterns are generated according to firing sequences described in the hypothetical diagram, and those patterns induce global states to have a chain like relationship. If global states are repeated having a sequence, the bound gait is realized. That is, in the global state i , each links of a robot are controlled by neuron patterns to make the robot be faced with the next, $i + 1$ global state.

However, if the sequence of firing of neurons in one global state is changed, the motion of the system is also changed. So determining global states to reappear a target motion is not easy. We propose two guidelines for determining global states. The first one is that global states must involve all motions of the target cyclic motion. If some motions are not included in global states, the target motion never can be reappeared. The second one is that reducing a number of global states makes the neuron pattern be accurate, and accurate patterns produce a better performance.

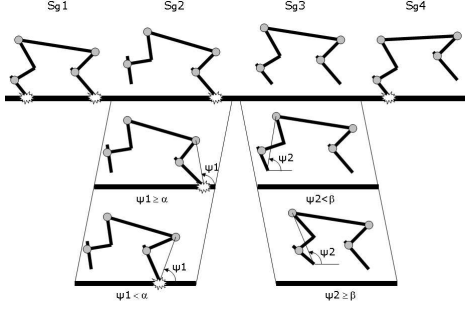


Fig. 2. Hypothetical global states.

	Sg1	Sg2	Sg3	Sg4
1				
2				
5				
6				
9				
10				
13				
14				

Fig. 3. Hypothetical Diagram. The number of the left column is the number of neuron. This diagram shows the sequence of firing of neurons in each global state.

2.3. Global State Dependent Neuron Network

We constructed the basic neuron network, and this network is good through a running control. Now, we construct flexible neuron networks depend on each global state. The network is available on one global state. So, if the global state is changed, the network between neurons is regenerated based on the hypothetical diagram. Equation (3) shows a mathematical expression of the network, and connections make neurons have a firing sequence. There are three considerations about determining the connecting weight, w , the target neuron that will be connected, and global states that make connections are available.

$$\begin{aligned}
Q_1 &= w_1 y_9 (+Sg2 + Sg3 + Sg4) + w_1 y_{10} (-Sg1) \\
&+ w_2 y_5 (-Sg1 + Sg2 + Sg4) + w_2 y_6 (+Sg3) \\
&+ w_1 y_{13} (-Sg1 + Sg2 + Sg4) + w_1 y_{14} (+Sg3) \\
Q_5 &= w_1 y_9 (+Sg2 - Sg3 + Sg4) + w_1 y_{10} (+Sg1) \\
&+ w_2 y_1 (+Sg2 - Sg3 + Sg4) + w_2 y_2 (+Sg1) \\
&+ w_1 y_{13} (+Sg1 + Sg2 + Sg4) + w_1 y_{14} (-Sg3) \\
Q_9 &= w_1 y_1 (+Sg2 + Sg3 + Sg4) + w_1 y_2 (-Sg1) \\
&+ w_1 y_5 (-Sg1 + Sg2 + Sg4) + w_1 y_6 (+Sg3) \\
&+ w_1 y_{13} (-Sg1 + Sg2 + Sg4) + w_1 y_{14} (+Sg3) \\
Q_{13} &= w_1 y_9 (+Sg2 - Sg3 + Sg4) + w_1 y_{10} (-Sg1) \\
&+ w_1 y_1 (+Sg2 - Sg3 + Sg4) + w_1 y_2 (+Sg1) \\
&+ w_1 y_5 (+Sg1 + Sg2 + Sg4) + w_1 y_6 (-Sg3) \\
Q_i &= -Q_{i-1}, Q_{i+1} = Q_{i-1}, Q_{i+2} = Q_i. \\
&(i = 2, 6, 10, 14)
\end{aligned} \tag{3}$$

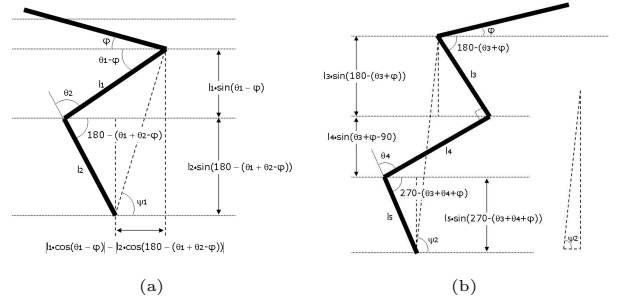


Fig. 4. (a) The global angle of front legs. This angle is a criteria for the ‘global state and posture dependent neuron input’ (b) The global angle of hind legs.

2.4. Global State and Posture Dependent Neuron Input

In hypothetical diagram, the sequence of firing of neuron 1 and 2 is reversed in the global state 2. Similarly, the sequence of neuron 5 and 6 is changed in the global state 3. Moreover, in global state 1 and 4, some neuron inputs are added to each neuron model to modulate a magnitude of neuron output patterns. The ultimate end of these sequence change is that a robot is faced with the next global state to maintain some sequences of global states.

- Inverting the pattern of paired neurons in a neural oscillator during a global state.
- Modulating the magnitude of neurons in a global state.

The second one can be achieved by adding an input to the neuron model, and the first one can be achieved by increasing this input enough to bring about the reverse turn of neurons in a neural oscillator.

Now, we define two angle criteria what we call the global angles, ψ_1 and ψ_2 , for a quadruped robot model. These angles are obtained from joint angles and body pitch angles. In front legs, the global angle, ψ_1 is calculated by geometrical information shown in Fig. 4(a) and (4).

$$\psi_1 = \tan^{-1} \frac{l_1 \sin(\theta_1 - \phi) + l_2 \sin(\pi - \theta_1 - \theta_2 + \phi)}{l_1 \cos(\theta_1 - \phi) - l_2 \cos(\pi - \theta_1 - \theta_2 + \phi)} \tag{4}$$

In hind legs, the global angle, ψ_2 is calculated by geometrical information shown in Fig. 4(b) and (5).

$$\psi_2 = \tan^{-1}(A/B) \tag{5}$$

where

$$\begin{aligned}
A &= l_3 \sin(\pi - \theta_3 - \phi) + l_4 \sin(\theta_3 + \phi - \pi/2) \\
&+ l_5 \sin(3\pi/2 - \theta_3 - \theta_4 - \phi) \\
B &= l_4 \cos(\theta_3 + \phi - \pi/2) - l_3 \cos(\pi - \theta_3 - \phi) \\
&- l_5 \cos(3\pi/2 - \theta_3 - \theta_4 - \phi)
\end{aligned}$$

2.4.1 Neuron Inputs About Front Legs

In front legs, the sequence of an activation between paired neurons in a neural oscillator is reversed by increasing the gain of the neuron input, q_1 , during the global state 2. The physical meaning is;

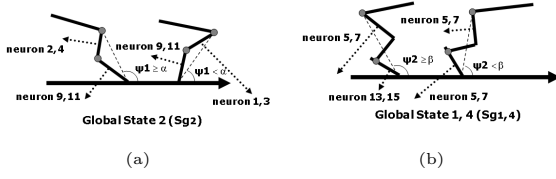


Fig. 5. (a) Physical meaning of neuron input (front legs) and (b) Physical meaning of neuron input (hind legs).

- If the front foot contacts with an environment (e.g. the global state 2) and the global angle, ψ_1 , is larger than α , a fore hip joint should be flexed toward the body with an absorption of the impact force until the global angle is smaller than α .

- If the contact is kept (e.g. the global state 2) and the global angle, ψ_1 , is smaller than α , a fore hip joint should be extended strongly to make a trust force.

Fig. 5(a) shows the physical insight of this law. The law is available with extension of fore knee joints only during the global state 2. Equation (6) shows neuron inputs, and the gain, q_1 , is 10.0.

$$\begin{aligned} S_1 &= -Sg2 \cdot q_1 \cdot 1(\psi_1 - \alpha) \\ S_2 &= -S_1, S_3 = S_1, S_4 = S_2 \end{aligned} \quad (6)$$

where

$$1(x) = \begin{cases} 1 & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

2.4.2 Neuron Inputs About Hind Legs

In hind legs, there are three global states to consider.

- Global state 3 (flight state)
- Global state 1, 4 (hind leg contacting states)

In the hypothetical diagram, the neuron 6(8 at right leg) is activated during the global state 3. This means that if a flight time increase, then hind legs are folded to the body, and running is failed. So neuron inputs control a direction of hind hip joints by using the criteria, ψ_2 , with respect to β . Equation (7) shows the law during the global state 3.

$$\begin{aligned} S_5 &= +Sg3 \cdot q_1 \cdot 1(\psi_2 - \beta) \\ S_6 &= -S_5, S_7 = S_5, S_8 = S_6 \end{aligned} \quad (7)$$

When hind legs are contacting with an environment, the global state 1 and 4 can be occurred. In this case, neuron inputs are added to the neuron model to modulate an intensity of the control torque. So the hind leg's trust force which is appropriate to run can be obtained. Fig. 5(b) and (8) show the physical meaning and its mathematical expression. Comparing with (1), in (9), Q_i is added to form networks between neurons depending on global states, and S_i is added to modulate a magnitude of neuron patterns and to reverse the activation sequence of neurons in a neural oscillator unit depending on global states. Both q_2 and q_3 are 3.0.

Table 2. Contact and non-contact gain values

	$j = 1$	2	5	6	9	10	13	14
p_j^c	450	250	350	100	500	130	600	120
p_j^{nc}	110	120	5	320	40	130	0	100

$$\begin{aligned} S_5 &= (+Sg1 + Sg4) \cdot q_2 \cdot (\psi_2 - \beta) \\ S_6 &= -S_5, S_7 = S_5, S_8 = S_6 \\ S_{13} &= (+Sg1 + Sg4) \cdot q_3 \cdot (\beta - \psi_2) \\ S_{14} &= -S_{13}, S_{15} = S_{13}, S_{16} = S_{14} \end{aligned} \quad (8)$$

$$\begin{aligned} Tr_i \cdot \dot{u}_i(t) &= -u_i(t) + \sum_{j=1}^{16} a_{ij} \cdot y_j(t) \\ &\quad + u_0 - b \cdot f_i(t) + Q_i + S_i \\ Ta_i \cdot \dot{f}_i(t) &= -f_i(t) + y_i(t) \\ y_i(t) &= \max(0, u_i(t)), \quad i = 1 \sim 16 \end{aligned} \quad (9)$$

2.5. Joint Torque Input

2.5.1 Torque Pattern Generation

neuron patterns generated by above rules have a potential to make the quadruped robot run. However, the magnitude of neuron patterns is not enough to control links of the robot model. So we amplify neuron patterns based on a condition of contact as (10).

$$\begin{aligned} T_i &= [p_i^c \cdot (S_k)_{on} + p_i^{nc} \cdot (S_k)_{off}] \cdot y_i, \quad i = 1 \sim 16 \\ p_j^c &= p_{j+2}^c \\ p_j^{nc} &= p_{j+2}^{nc} \end{aligned} \quad (10)$$

where

$$k = \begin{cases} 1(\text{left front foot}) & (i = 1, 2, 9, 10) \\ 2(\text{right front foot}) & (i = 3, 4, 11, 12) \\ 3(\text{left hind foot}) & (i = 5, 6, 13, 14) \\ 4(\text{right hind foot}) & (i = 7, 8, 15, 16) \end{cases}$$

In (10), $(S_k)_{on}$ and $(S_k)_{off}$ are contact and non-contact information from feet, respectively. Gains expressed by j are shown in Tab.2. These gain values are determined by the progress as follows:

- Determining non-contact gains to reconstruct a running motion
- Determining contact gains
 - Determining gains of hind legs
 - Determining gains of front legs

2.5.2 Joint Torque Input

Control torque inputs are obtained as (11). Tna_m means a torque to maintain neutral positions of each joint using PD controller. Because the central pattern generated by neuron networks has no consideration about a gravity effect. Tab.3 shows the identification of joints controlled by joint torque input, T_m .

Table 3. Identification of joints controlled by joint torque input

m	Joint
1	Left front hip joint
2	Right front hip joint
3	left hind hip joint
4	right hind hip joint
5	left front knee joint
6	right front knee joint
7	left hind ankle joint
8	right hind ankle joint

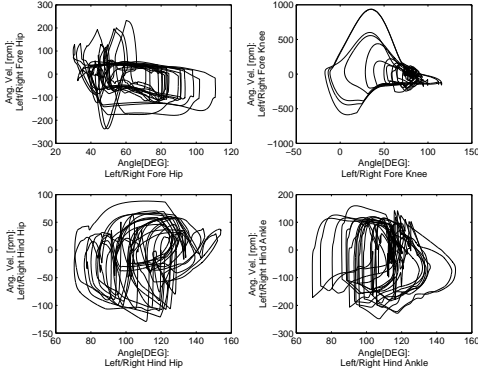


Fig. 6. Phase Portrait(flat terrain).

$$T_m = Ttp_m + Tna_m, \quad m = 1 \sim 8. \quad (11)$$

where

$$Ttp_m = T_{2m} - T_{2m-1}.$$

3. Simulation Results

In the simulation study, the MATLAB is used to control the robot, and the dynamic simulator, DADS9.6 is used as a 3D dynamic environment. Total length of the robot model is about 50cm, and total weight and height are about 10kg and 30cm respectively.

An up and down sloped terrain is considered as an unknown environment to validate robustness of the proposed single frame work controller, and the proposed training-free controller makes the quadruped robot have an adaptation ability to an environment.

3.1. Running on a Flat Terrain

The robot shows a stable running as Fig. 6, even if the motion cycles throughout running are not exactly the same. The reason for this result is that global states have the sequence. In Fig. 7, global states show two sequences:

1. Sequence A: Sg4 \rightarrow Sg3 \rightarrow Sg2 \rightarrow Sg1 \rightarrow Sg4
2. Sequence B: Sg4 \rightarrow Sg3 \rightarrow Sg2 \rightarrow Sg3 \rightarrow Sg4

The global state 1 is substituted for the global state 3 in sequence B. And, the summation of global states are 1. This means that all motions are included in global states.

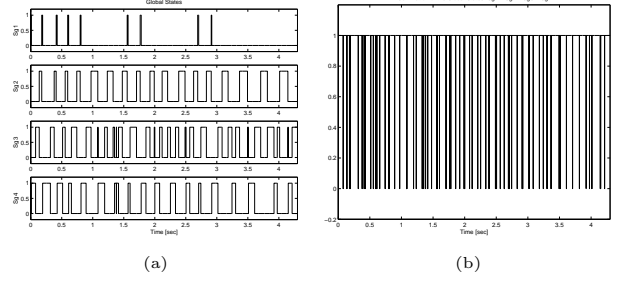


Fig. 7. (a) Global state and (b) summation of the global states(flat terrain).

Fig. 8 shows the forward speed and body pitch angle. In this figure, the magnitude of the body pitch angle has an inverse proportion to the forward speed.

The control law does not use a forward velocity as the control parameter, so it can not tell us what is the value of the control parameter to satisfy a specific velocity. In other word, we can just expect that the forward speed is in proportion to the constant, u_0 , because the magnitude of the neuron pattern is in proportion to the u_0 .

3.2. Running on the Combined Terrain

The angles of up and down slopes are 4° . Fig.9 shows this geometry of the combined terrain.

In dynamic simulation result, the robot went up an up slope at 1.1 sec, then arrived at the flat floor at 2 sec. At 3 sec, the robot started to go down a slope, then arrived at the lower floor at 4 sec as Fig. 10(a), and global states show sequences which consist of the sequence A and B mentioned above.

4. Conclusions

In the proposed control law, neuron patterns were generated according to firing sequences described in the hypothetical diagram, and those patterns induced global states to have a chain like relationship. And if global states are repeated having some sequences, the bound gait is realized. For general use of global states, we proposed guidelines for determining global states which describes the cyclic motion. And these tips are available on any kind of cyclic motions as a target motion.

The central pattern based approach made the controller have a single framework, that is, there is no controller change according to states or support modes, and the global state

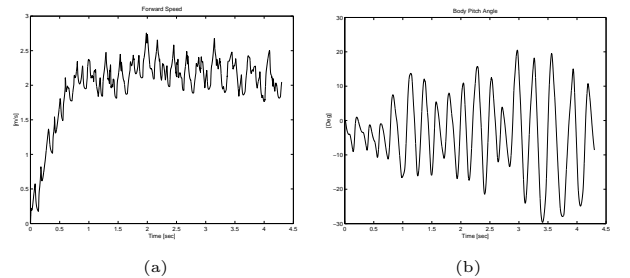


Fig. 8. (a) Forward speed(m/s) and (b) Body pitch angle(deg).

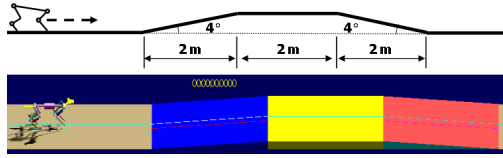


Fig. 9. Geometry of the combined terrain.

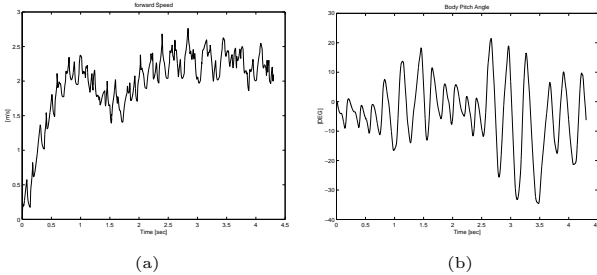


Fig. 10. (a) Forward speed(m/s) and (b) Body pitch angle(deg) on the combined terrain.

dependent neuron network induced adaptation ability to an environment. And calculation burden was resolved by extracting dynamic equations from the control loop. This makes calculation efficiency good obviously comparing with the control method based on the inverse dynamics.

The 3D dynamic simulation was performed on different terrain conditions, but parameters in the proposed control law do not need to be changed according to the terrain. And the simulation results showed the proposed single framework controller is robustly performed in an unpredictable sloped terrain without training.

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