

Improved Exploration Algorithm Using Reliability Index of Thinning Based Topological Nodes

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Abstract: For navigation of a service robot, mapping and localization are very important. To estimate the robot pose, the map of the environment is required and it can be built by exploration or SLAM. Exploration is the fundamental task of guiding a robot autonomously during mapping such that it covers the entire environment with its sensors. In this paper, an efficient exploration scheme based on the position probability of the end nodes of a topological map is proposed. In this scheme, a topological map is constructed in real time using the thinning-based approach. The robot then updates the position probability of each end node maintaining its position at the current location based on the Bayesian update rule using the range data. From this probability, the robot can determine whether or not it needs to visit the specific end node to examine the environment around this node. Various experiments show that the proposed exploration scheme can perform exploration more efficiently than other schemes in that, in most cases, exploration for the entire environment can be completed without directly visiting everywhere in the environment.

Keywords: exploration; topological map; thinning process; position probability

1. INTRODUCTION

There are two key elements for an autonomous mobile robot; mapping and localization. Mapping is the task of modeling a robot's environment and localization is the process of determining the position and orientation of a robot with respect to the global reference frame. A map is required for several navigation tasks, for example, localization, path planning, etc. Two major paradigms have been used for mapping the indoor environment: an occupancy grid map and a topological map. The former can produce an accurate metric map in a relatively simple manner, but it requires a large amount of memory and is inefficient. The latter, on the other hand, provides more efficient and compact Voronoi diagram-like map requiring much less memory, but is difficult to apply to accurate localization due to lack of information [1].

To map an unknown environment, the robot requires some strategy which guides it autonomously to cover the environment, which is called *exploration*. The exploration scheme serves as a key element to autonomous SLAM (Simultaneous Localization And Mapping). The requirements for exploration are its completeness and efficiency. The completeness means that it should cover the entire environment without missing any portion of the environment. The efficiency means how fast the robot can finish the task of mapping by minimizing the travel distance to cover the entire environment.

Several algorithms for exploration have been proposed so far. In the early days of mobile robot navigation, mapping was performed by sonar sensors with the feature-based localization using the beacons in [2]. But at that time the exploration scheme was not developed and mapping was carried out manually. The integrated exploration containing all of map building, localization and motion control was also studied in [3]. In this paper the next destination is evaluated by multiple utility functions and selected. Exploration using wall-following was proposed in [4], but this approach was inefficient and had many limitations. Choset proposed a T-SLAM algorithm which used both the GVG-based topological map and the annotated geometric landmarks in the

map [5]. It worked well if there were sufficient obstacles inside the sensor range of the robot. But its exploration scheme was not efficient because the robot should reach boundaries and the exploration was complete when all meet points had no unexplored edges associated with them. In the frontier-based exploration, a robot navigated around the boundary between the unexplored and known space in the grid map [6] and it has been the most popular exploration scheme adopted by many researchers. However, little research has been done to improve the efficiency of exploration by observing the state of the environment.

In this paper, the improved approach to building a thinning-based topological map is presented. In the previous thinning-based topological map [7], a local occupancy grid map was constructed using the range data. From this map, a local topological map was then built in real time by applying the thinning process and incrementally updated to a global topological map. Based on this thinning-based topological map, a new method is developed to estimate how much of the environment is explored by observing the change in end node positions in real time.

To this end, the position probability of each end node maintaining its position at the current location based on the Bayesian update rule using the range data is computed every sampling period. Since the higher position probability means that the environment around the corresponding node is examined thoroughly enough for precise mapping, exploration is conducted only for the nodes whose probability is below a certain threshold. With this proposed scheme, the robot does not need to explore the entire environment to precisely model the environment, so fast and efficient exploration can be achieved.

This paper is organized as follows. Section 2 reviews how to build a thinning-based topological map and section 3 introduces the concept of position probability of an end node and deals with how to efficiently explore the unknown environment using the position probability. Experimental results are shown in section 4 and finally in section 5 conclusions are drawn.

2. REVIEW OF THE THINNING-BASED TOPOLOGICAL MAP

2.1. Edge Generation

In the feature-based topological map, the environment is modeled by a set of geometric primitives such as edges and nodes. It has several advantages such as compactness, fast computation, natural expression to human, and so on [1]. A topological map, however, is not appropriate for localization which requires comparison of the current map with the reference map because it has only limited feature information compared to a grid map.

To build a topological map using range sensors, a Voronoi diagram is commonly used. But it is complex and difficult to apply to arbitrarily shaped objects. In building a topological map, therefore, a thinning method was proposed in [9], which needs simpler computation than the Voronoi diagram, but can show similar performance.

A thinning method is one of the popular image processing algorithms, which have been used to detect the skeleton of images. Fig. 1 illustrates the concept of thinning. The objects on the left can be described satisfactorily by the structure composed of connected lines (i.e., ‘T’ shape drawn with thin lines on the right). Note that connectivity of the structure is still preserved even for representation with thin lines. In the case of mobile robots, the connected lines are used as paths on which a robot navigates without colliding with other objects.

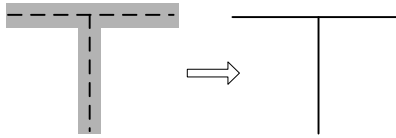


Fig. 1 Concept of thinning.

The detailed algorithm of the thinning process can be referred to in [8]. As shown in Fig. 2, the free space to which the thinning process is applied is selected. Then the free space continued to be contracted from both the outside of the objects and the inside of the wall boundary. Thinning process is repeated until the skeleton corresponding to the thinnest line for the free space is extracted.

The thinning-based topological map is constructed as follows. The robot collects the range data by scanning the environment using a laser rangefinder. Since the scanning rate is about 5 Hz and the robot navigates slowly, each cell is likely to be scanned several times. The occupancy probability for each cell is then updated based on the Bayesian update formula. This probabilistic approach to building a local grip map enhances the confidence of the underlying grid map for the local topological map. At each sampling instant, based on the range data, the local grid map and the subsequent local topological map is built. The local topological map even for the same space is constantly changing as the underlying grid map is updated. Since the thinning-based topological map building can be executed fast, the topological map can be constructed on the real-time basis.

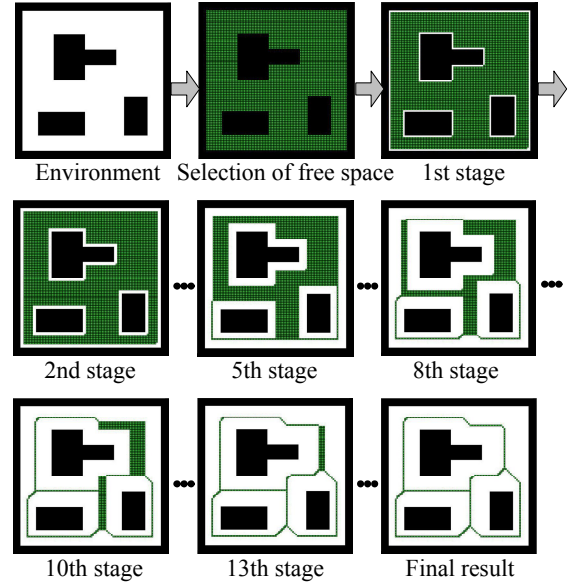


Fig. 2 Edge generation of a thinning-based topological map for given environment (Simulations).

2.2 Node Generation

After the edges are extracted through the thinning process, three types of nodes can be identified from these edges. Fig. 3 explains the node extraction algorithms. The first is the *branch node* (called a “meet point” at the GVG) at which more than three edges meet. The second is the *end node* which represents the end of each edge. These two types of nodes are generated by means of counting the number of cells composing the edges. As shown in Fig. 3(a) and (b), if the cell has only one neighboring cell, this cell is the *end node*. If a cell has more than three neighboring edges, this happens to be the *branch node*. The third is the *corner node* and it is generated when the edge slope varies significantly and the obstacle is placed on the straight line connecting the two nodes. In Fig. 3(c), no obstacle exists on the line between nodes A and B and thus no corner node is generated. In Fig. 3(d), however, the line connecting the two nodes is blocked by the obstacle and thus the corner node is generated at the cell that has the maximum distance from the line.

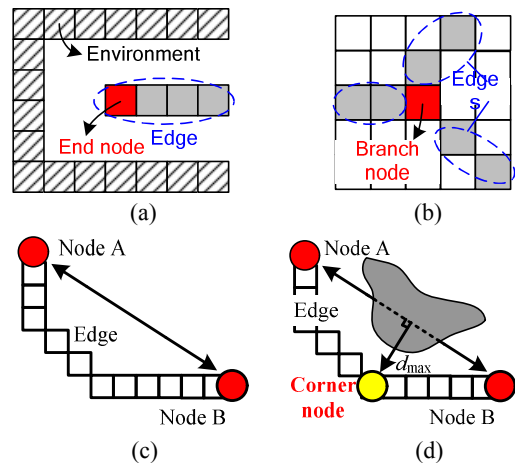


Fig. 3 Node generation in a thinning-based topological map for given environment (Simulations).

Fig. 4 shows an example of the thinning-based topological map. Edges and three types of nodes are illustrated. The end nodes represent the corner of the environment like the dead end of the hallway and the branch nodes indicate the area in which the environment is divided into several parts due to obstacles. If the topological edges are used as a path for navigation, the end nodes mean the end of the path and the branch nodes mean the intersection at which more than three paths meet. The corner nodes represent the curved area of the environment and are required to provide the rough sketch of an edge since only the coordinates of the nodes and their connectivity are stored in the final topological map without the detailed information on the edges. For example, without introducing the corner node N_3 in Fig. 4, only the information on the connectivity of nodes N_1 and N_2 cannot generate edge A which can detour the object. With corner node N_3 , a new edge similar to the edge A can be reconstructed by combining the line between node N_1 and N_3 and the line between node N_2 and N_3 . Hence, with all types of nodes and edges, the topological information enough for mobile robot navigation can be obtained as shown in Fig. 4.

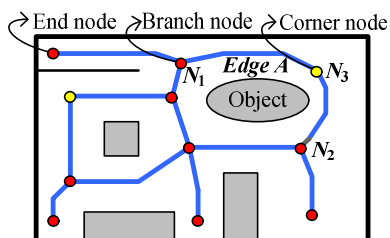


Fig. 4 Three types of nodes in a thinning-based topological map.

3. POSITION PROBABILITY BASED EXPLORATION

3.1 Position Change of End Nodes

The nodes of a thinning-based topological map have their own characteristics. For example, the positions of the branch nodes at which more than three edges meet are relatively robust to a change in environment and the sensing methods, so they can be used as a landmark for localization [9]. On the other hand, the position of the end nodes representing the end of an edge is likely to change according to the degree to which the environment is sensed by the range sensor mounted on the robot. That is, it is related to visibility of the sensor. Therefore, the observation of the end node position may indicate how much of the environment is explored.

Fig. 5 shows the topological map constructed in real time when the robot navigated through the unknown environment. In the figure, the circle with the red mark denotes a mobile robot with the red mark pointing to the robot's heading. In Fig. 5(a), the robot scanned the unexplored frontal area using the laser range scanner capable of providing 181 range readings with a resolution of 1° . Based on these range data, the local grid map and then the local thinning-based topological map were constructed as shown in the figure. In this initial topological map shown in Fig. 5(a), 1 branch node, 3 end node, and 1 corner node were created. Consider node A in the figure for instance. No information on the area to the left of node A was available with the current sensor location. However, as the robot traversed towards node A and the unseen area around node A was exposed to the range sensor, node A continued to change its position (through (b) and (c)) until the

robot thoroughly detected the region around the dead end of the hallway. Note that the branch nodes maintained their position unlike the end nodes.

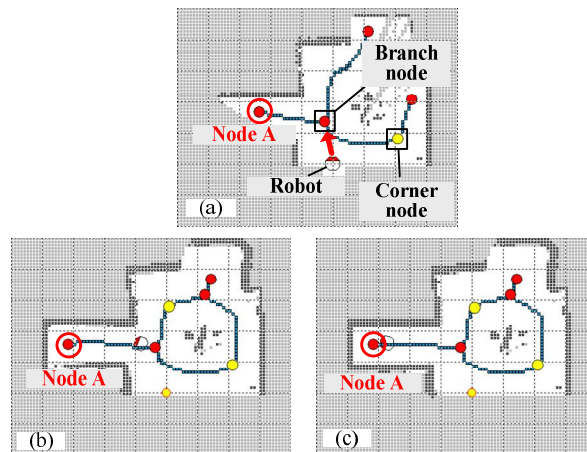


Fig. 5 Change in end node position during exploration of dead end of hallway.

Fig. 6 shows the situation different from Fig. 5. Since the newly sensed area was not the dead end of a hallway and a new hallway was found, the end node sprawled out into this hallway as the robot approached this end node. Consequently, if the position of the end node varies no longer as the robot approaches the end node, we can determine that the environment around the end node is almost fully explored.

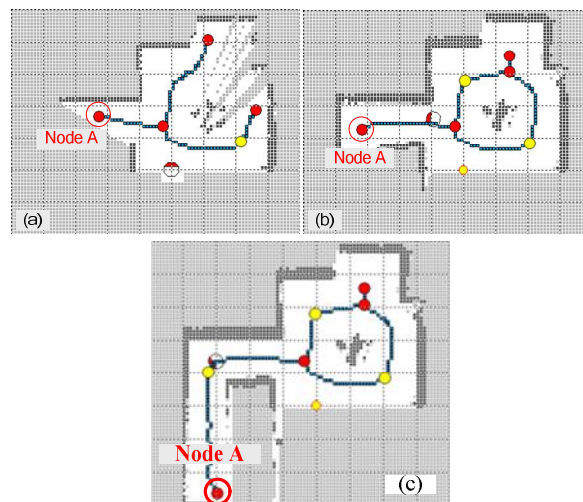


Fig. 6 Change in end node position during exploration of connecting area of two hallways.

3.2 Position Probability Based Exploration

One popular exploration method is to identify the boundary between the known and unknown areas and to visit the nearest unknown area in succession. Especially, in the topological exploration based on the GVG or the thinning-based topological map, the environment can be fully explored by visiting all end nodes generated during the map construction [5]. But in this case exploration may not be efficient because the robot is obliged to visit all end nodes. To enhance the efficiency of exploration, this paper proposes a new method to efficiently explore the environment using the information on

the topological nodes updated in real time.

Thorough examination of the environment from the node under consideration guarantees accurate modeling of the environment, but it often leads to inefficient and time-consuming exploration. If the mobile robot can model the environment around the end node from a distance without closely approaching the node, the exploration time can be reduced. However, it is not easy to determine whether the close examination has been conducted or not. To this end, we define the *reliability index* that indicates how much of the environment around the end node under consideration is explored by investigating the behavior of nodes.

In this context, the reliable node position means that the end node under consideration will stay at the current location after the exploration around this node is completed. For example, if the position of the end node does not alter even after scanning the environment around the end node, then the position probability of the end node staying at this current position is regarded as 1.

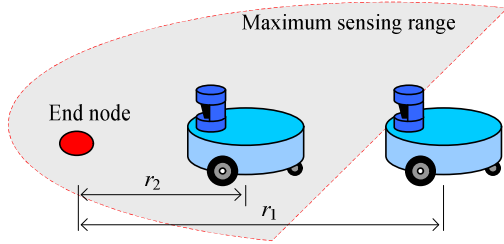


Fig. 7 A robot can scan the environment around the node in different distances.

The first reliability index is associated with the distance between the node and the robot. Obviously, the closer the robot approaches the end node, the more thorough exploration can be conducted, thereby resulting in an increase in the reliability of the node position. The *distance reliability index* for end node n is represented by $D_n(\mathbf{X}_{node,t}, \mathbf{X}_{robot,t})$, where $\mathbf{X}_{node,t}$ and $\mathbf{X}_{robot,t}$ are the absolute positions of end node n and the robot at time t relative to the global reference frame, respectively. It is assumed that the distance index of the end node staying at the current location is 1.0 when the distance between the robot center and the end node is less than 0.5m (i.e., the distance between the node and the laser scanner is about 0.3m in consideration of the robot radius of 0.2m), because the environment is highly likely to be closely examined at this distance. If the distance is greater than a certain threshold (in this case, 4.5m), then it is assumed that the robot is on the verge of reasonable modeling of the environment, thus assigning the index of 0.5. Between these two boundaries, the distance index tends to decrease in inverse proportion to the distance between the node and the robot. In summary, the distance reliability index for end node n is defined as

$$D_n(\mathbf{X}_{node,t}, \mathbf{X}_{robot,t}) = \frac{4.0}{r + 3.5} \quad (1)$$

where

$$r = \begin{cases} 0.5, & \text{if } (\min_i \|\mathbf{X}_{node,i} - \mathbf{X}_{robot,i}\| < 0.5) \\ \min_i \|\mathbf{X}_{node,i} - \mathbf{X}_{robot,i}\|, & \text{else} \end{cases} \quad (i=1, \dots, t) \quad (2)$$

In Eq. (2), r represents the minimum of the distances experienced during the operation up to time t . It is also noted that the end node out of the scanning range (180°) or blocked by obstacles is excluded in the computation of Eq. (2).

The second reliability index is associated with the angle between sensing positions. It can be easily understood that more information on the environment around the end node can be collected when the node is scanned from various angles. Let us define the angle difference as

$$\alpha_{\max} = \max_{i,j} \left| \angle(\mathbf{X}_{node,i} - \mathbf{X}_{robot,i}) - \angle(\mathbf{X}_{node,j} - \mathbf{X}_{robot,j}) \right| \quad (i, j = 1, \dots, t) \quad (3)$$

where $\angle(\mathbf{X}_{node,i} - \mathbf{X}_{robot,i})$ is the angle of the vector from the robot center to the end node measured counterclockwise relative to the X axis of the global reference frame. It is also noted that the end node out of the scanning range (180°) or blocked by obstacles is excluded in the computation of Eq. (3). Note that the angle difference α_{\max} is the maximum of the angle differences $|\alpha_1 - \alpha_2|$, $|\alpha_2 - \alpha_3|$, and $|\alpha_3 - \alpha_1|$ in the case of Fig. 8, so $\alpha_{\max} = |\alpha_3 - \alpha_1|$.

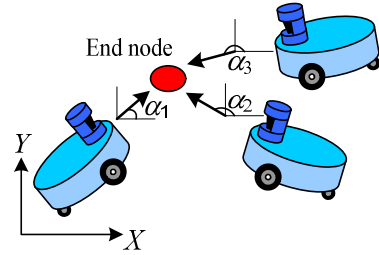


Fig. 8 A robot can scan the environment around the node from different angles.

It is likely that the *angle reliability index* of the end node staying at the current location gets higher when the robot sees the end node from various angles. Therefore, the angle reliability index for end node n , A_n , is defined by

$$A_n(\mathbf{X}_{node,t}, \mathbf{X}_{robot,t}) = \begin{cases} \frac{\alpha_{\max}}{90} & (\alpha_{\max} \leq 90^\circ) \\ 1 & (\alpha_{\max} > 90^\circ) \end{cases} \quad (4)$$

Note that the angle index of Eq. (4) is so designed that $A_n = 1$ when $\alpha = 90^\circ$ and $A_n = 0$ when $\alpha = 0^\circ$. Since the laser scanner has a scanning range of 180° , $\alpha_{\max} = 180^\circ$ means that the neighboring environment of the end node in the range of 360° is scanned. However, the end nodes are usually placed in a corner as shown in Fig. 9, the position of the end node can be determined even when $\alpha_{\max} = 90^\circ$.

End nodes can be generated in two cases as shown in Fig. 9. In (a), the end node is generated at a corner where two walls meet. In this case, the robot does not need to visit the end node because examining the environment around the node from various angles (even at a great distance) is sufficient to precisely model it. In (b), the end node is generated at the dead end of a narrow hallway. It is difficult to scan the environment from various angles, so the robot needs to approach the end node sufficiently closely. In conclusion, the angle index is more preferable in case (a), while the distance index in case (b).

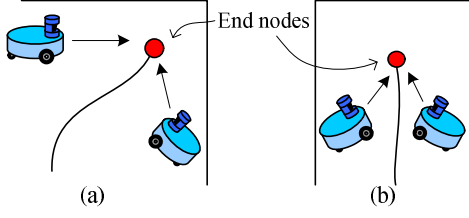


Fig. 9 Two cases in which end nodes are generated.

In order to cover both cases shown in Fig. 9, the overall reliability index, R_n , is defined as

$$R_n(\mathbf{X}_{node,t}, \mathbf{X}_{robot,t}) = \max(D_n, A_n), \quad (5)$$

where n is the node number. Using the reliability index computed in Eq. (5), the *position probability* for every end node of a topological map can be calculated. The position probability means the possibility of the end node maintaining its position at the current location after completion of the exploration. The exploration task is executed in the unknown environment, so the nodes and edges of a topological map continue to change in position and shape, respectively. In some cases, the nodes and edges appear and disappear as the exploration progresses. Therefore, the position probability of end node n should be updated by the following Bayes' update formula:

$$P_n[s(E_n) = \text{rel} | \{\mathbf{X}\}_{n,t+1}] = \frac{p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{rel}] \cdot P_n[s(E_n) = \text{rel} | \{\mathbf{X}\}_{n,t}]}{\sum_{s(E_n)} p_n[\mathbf{X}_{n,t+1} | s(E_n)] \cdot P_n[s(E_n) | \{\mathbf{X}\}_{n,t}]}, \quad (6)$$

where E_n represents end node n , $s(E_n)$ is the state of E_n which is either *reliable* or *unreliable*, P_n is the position probability indicative of how reliable the current position of end node n is. Hence, the state of " $s(E_n) = \text{reliable}$ " means that the end node maintains its current location after exploration is completed. In Eq. (6), $\mathbf{X}_{n,t}$ denotes the position of both the node n and the robot at time t (i.e., $\mathbf{X}_{node,t}$ and $\mathbf{X}_{robot,t}$ for notational convenience) and $\{\mathbf{X}\}_{n,t}$ the position data up to time t , and $\mathbf{X}_{n,t+1}$ is the newly measured position data at time $t+1$. The denominator of Eq. (6) can be rearranged as

$$\begin{aligned} \sum p_n[\mathbf{X}_{n,t+1} | s(E_n)] \cdot P_n[s(E_n) | \{\mathbf{X}\}_{n,t}] = \\ p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{rel}] \cdot P_n[s(E_n) = \text{rel} | \{\mathbf{X}\}_{n,t}] \\ + p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{unrel}] \cdot P_n[s(E_n) = \text{unrel} | \{\mathbf{X}\}_{n,t}] \end{aligned} \quad (7)$$

For computational purpose, it is assumed that

$$p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{unrel}] = 1 - p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{rel}] \quad (8)$$

$$P_n[s(E_n) = \text{unrel} | \{\mathbf{X}\}_{n,t}] = 1 - P_n[s(E_n) = \text{rel} | \{\mathbf{X}\}_{n,t}] \quad (9)$$

On the other hand, $p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{rel}]$ is computed using the relation Eq. (5) by

$$p_n[\mathbf{X}_{n,t+1} | s(E_n) = \text{rel}] = R_n(\mathbf{X}_{node,t+1}, \mathbf{X}_{robot,t+1}) \quad (10)$$

Note that the initial position probability of 0.5 is assigned to all end nodes, so $P_n[s(E_n) = \text{rel} | \{\mathbf{X}\}_{n,1}] = 0.5$.

During exploration, the position probabilities of the end nodes within the scanning range of the laser scanner at the

current time are computed by Eq. (6). The robot visits only the end nodes whose position probability is lower than a certain threshold (0.9 in this research) until all end nodes have the position probability greater than the threshold. If the edges of a topological map are chosen as an exploration path, the robot can safely navigate the unknown area while avoiding static obstacles without additional path planning.

In this research, the A* search method is used to determine the target end node to visit next. Once the target node is selected, the robot explores the environment along a topological edge on which the end node exists. As mentioned before, during navigation to the target node the robot sees other end nodes within its scanning range, and the probability of all these nodes are updated although these are the current target nodes. Using this exploration strategy based on the position probability of the end nodes, the robot can explore the unknown environment more efficiently and rapidly than the previous exploration methods.

4. EXPERIMENTS

The proposed exploration scheme has been tested on various environments using Pioneer 2DX. One laser scanner (SICK LMS200) with a scan range of 180° was installed with the scanning area facing forward. Fig. 10 shows the exploration process. In the figure, each cell of the grid map is $10\text{cm} \times 10\text{cm}$ in size and the environment is $10\text{m} \times 10\text{m}$ in size. The figures beside each end node denote the end node number and its position probability computed by Eq. (6) at the current time. For example, 1(0.95) means that the position probability of end node 1 staying at the current location after complete exploration is 0.95. During exploration, the robot moved with a maximum velocity of 200mm/s , and the total time required was less than 2 minutes.

The robot was placed in the unknown environment at the beginning and it scanned its neighboring environment to construct a grid map with the range data. The topological map was then created by the thinning process as explained before. An initial value of 0.5 was assigned to each end node as its initial position probability. In Fig. 10(a), end node 1 was selected as a target node because the robot's heading was directed toward this node. Note that choice of an initial target node does not affect the exploration results. As the robot traversed toward end node 1 along the topological edge created as a result of topological map building, the position probability of end node 1 continued to increase. If it exceeds 0.9, then the end node is believed to maintain its position with confidence enough to guarantee that the robot does not need to visit this node to complete the modeling of its environment. In Fig. 10(b), therefore, the position probability was found to be 0.95, the robot started to head for another end node nearest end node 1 without directly visiting end node 1.

In Fig. 10(c), the robot was heading for end node 2. As the robot approached this node, the new environment which had not been visible in Fig. 10(b) was now detected, thereby causing the target node to keep moving as shown in Fig. 10(c), (d). Note that most environment of the room could be roughly modeled even at the time of Fig. 10(c) and the topological map continued to be updated in real time. Through Fig. 10(c), (d), (e), the probability of end node 2 did not increase enough to reach the threshold of 0.9, thus meaning that the probability of this node staying at that location was low. It turned out that end node 2 disappeared in Fig. 10(f), indicating that node 2 was not an end node. Now the robot headed for another end node 3 and stopped exploration after the position probability of end node 3 was found to be 0.99.

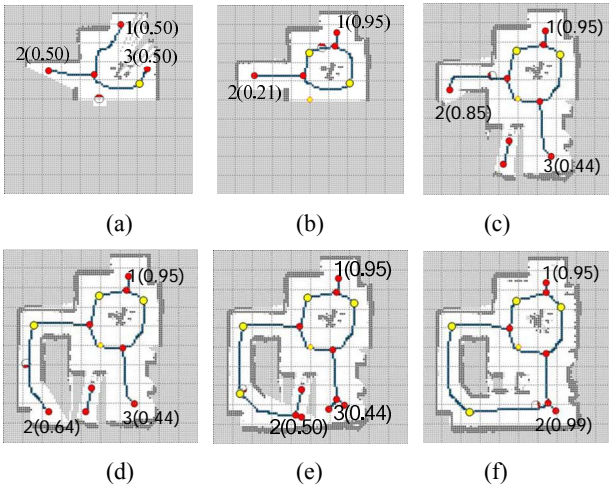


Fig. 10 Changes in end nodes and their probabilities during exploration.

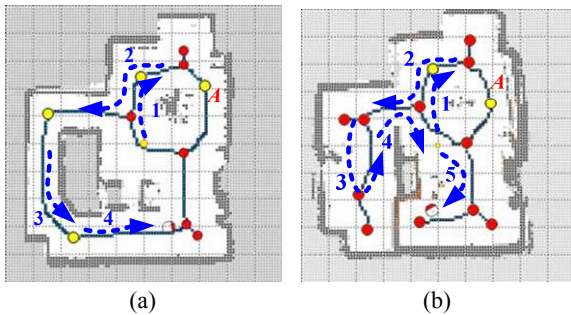


Fig. 11 Grid and topological maps created using the proposed exploration scheme and robot paths illustrated during exploration.

In Fig. 11(a) and (b), paths 1, 2, 3, 4 and 5 illustrates the actual paths along which the robot navigated in succession during exploration. As can be seen in the figures, the robot did not explore the entire environment, but the relatively accurate environment map was obtained. Note that the robot did not visit the right portion of the environment at all. In the Fig. 11, the topological loops were formed around the region *A* which consisted of desks and chairs. The exact shapes of the objects were not reflected in the grid map because the laser sensor scanned only the legs of the desks and chairs.

5. CONCLUSIONS

In this paper, an efficient exploration scheme based on the position probability of the end nodes of a topological map was proposed. With this scheme, the robot does not have to visit the entire environment to accurately model the unknown environment. The validity of this proposed exploration strategy has been verified by a series of experiments. From this research, the following conclusions are drawn.

1. The thinning-based approach can provide a simple but robust way of constructing a topological map in real-time.
2. By observing the position of end nodes of the topological map constructed in real-time, the degree of exploration can be determined. By updating the position probability of the end nodes, the robot does not have to visit the end nodes whose probabilities are greater than a predefined threshold to complete exploration.
3. The edge of a topological map can serve as a navigation path which is free of static obstacles and does not require

additional path planning for exploration of the unknown environment.

Currently, research on more efficient exploration strategy for the large-scale environment is under way. And more robust exploration algorithm that is reliable and useful in spite of some moving obstacles is also under way.

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