A SDINS Error Compensation Scheme Using Star Tracker

Jong-bin Yim*, Joon Lyou*, and You-chol Lim**

* Department of Electronics Engineering, Chungnam National University, Daejeon, Korea
(Tel: +82-42-823-3533; E-mail: jbiny@cnu.ac.kr)
**Department of Electronics, Korea Aerospace Research Institute, Daejeon, Korea
(Tel: +81-42-860-2594; E-mail: yclim1002@kari.re.kr)

Abstract: Since inertial sensor errors which increase with time are caused by initial orientation error and sensor errors (accelerometer bias and gyro drift bias), the accuracy of these devices, while still improving, is not adequate for many of today’s high-precision, long-duration sea, aircraft, and long-range flight missions. This paper presents a navigation error compensation scheme for Strap-Down Inertial Navigation System (SDINS) using star tracker. To be specific, SDINS error model and measurement equation are derived, and Kalman filter is implemented. Simulation results show the boundedness of position and attitude errors.

Keywords: SDINS, Star Tracker, Sensor fusion, Kalman filter

1. INTRODUCTION

The 1970s and 1980s have seen INS (Inertial Navigation System) dominate dead reckoning devices for ships, and planes. The accuracy of these devices is not adequate for many of today’s high precision, long duration sea, aircraft, and long-range flight missions because of the inertial sensor errors which increase with time are caused by initial orientation error, accelerometer bias error and gyro drift bias error. INS accuracy can be improved by augmentation with position or velocity updates from other navigation sensors. Although radio based schemes using GPS (Global Position System) have received much attention for providing this augmentation, an autonomous sensor is preferable because active sensors may be unavailable as a result of hostile attacks on, or reliability failures of radio based navigation aids during the critical phases of the mission. Much of the electro-optical machinery necessary to implement an effective inertial navigation augmentation system can also be used as a terminal or target sensor system. However, until now, the aided stellar inertial system has been hardly studied in domestic field because the sensor is so expensive and also does not fields of various applications. Therefore, in this study, we suggest SDINS (Strap-Down Inertial Navigation system) navigation error compensation schemes using each position and attitude information calculated by celestial navigation computer.

2. ELECTRO-OPTICAL SENSOR

The fundamental principle of the celestial navigation is to estimate the vehicle’s position and attitude of present time by comparison the celestial sphere constellation pattern information with the constellation pattern which is stored in star catalogue.

Figure 1 shows the information obtained by star tracker. It can be obtained azimuth and elevation angle with respect to earth centered inertial frame (ECIF: $X_1, Y_1, Z_1$), LOS vector with respect to star tracker coordinate frame (SCF: $X_s, Y_s, Z_s$). Position and attitude information are calculated by the celestial navigation computer.

From a moving vehicle, the measurement of the directions of two or more stars with respect to a vehicle-fixed coordinate system provides an instantaneous determination of the vehicle’s attitude with respect to earth centered frame. If there is also a determination of the direction of the local vertical (gravity vector) with respect to the same coordinate system, the vehicle’s attitude with respect to the local horizon system can be obtained. And additionally, if initial position is known, vehicle’s position with respect to the earth centered earth fixed (ECEF) frame can be obtained by the celestial navigation algorithm. The determination of the local vertical is not trivial for a moving vehicle and in general will require corrections for coriolis forces and geophysical deflection.

The local vertical provides the direction orthogonal to the geoid and, appropriately corrected, toward the center of the Earth. To obtain latitude and longitude, we need a time reference.

Figure 2 represents a simple composition of star tracker.

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Figure 2 represents a simple composition of star tracker.
Against star one it is gotten the data of \( p_s \) and the \( p_t \) from the CCD camera of the star tracker. If there is not a distortion of the lens, the image of the star is put at the straight line which passes by the center of the lens. Let \( \alpha \) is a angle between LOS vector and \( y_{ST} - z_{ST} \) plane, and \( \beta \) is a angle between \( z_{ST} \) axis and projected line of LOS vector with respect to \( y_{ST} - z_{ST} \) plane.

\[ \alpha = \tan^{-1}\left( \frac{p_s \cos \beta}{f} \right) \]  
\[ \beta = \tan^{-1}\left( \frac{p_t}{f} \right) \]  

And, it expresses the unit LOS vector of the star with function of \( \alpha \) and \( \beta \).

\[ u' = \begin{bmatrix} \sin(\alpha) \cos(\beta) \\ \cos(\alpha) \cos(\beta) \\ \sin(\beta) \end{bmatrix} \]

A star catalog is a stored data file of stars designated by name or ID containing data fields describing the star, the critical data for stellar-inertial navigation system being star position and magnitude. Star position is referenced to the vernal equinox and the celestial equator. The measured star positions in the Field-Of-View(FOV) of a star camera are identified using an appropriate star identification algorithm, and then the attitude and position are calculated, which finally determines the best estimate for the angular orientation and location of a vehicle.

We suggest SDINS error compensation schemes using attitude and position information calculated by the celestial navigation algorithm. In this study, these detail algorithms for calculating the attitude and position are omitted due to the out of focus. In this study, it is assumed that we just use the attitude and position information of celestial navigation.

3. SDINS ERROR COMPENSATION SCHEME

3.1 A compensation scheme using position information of star tracker.

The object of celestial navigation is the determination of the latitude and longitude of a vehicle at a specific time through the use of observations of the altitudes of celestial bodies. The chart-based approach translates each celestial altitude observation into a Line-Of-Position(LOP) on the surface of the earth. In principle, a series of observations defines a group of intersection LOPs, and this intersection represents the observer’s position fix.

Figure 3 shows the block diagram of the SDINS error compensation scheme using position information of star tracker.

3.2 A compensation scheme using attitude information of star tracker.

We also suggest another scheme using attitude information calculated by the celestial navigation algorithm. Attitude determination for star sensor requires the identification process of a proper orthogonal rotation matrix so that the measured observations in the star tracker frame equal to the reference frame observations in the star catalog mapped by the matrix into the sensor frame.

Figure 4 shows the block diagram of the SDINS error compensation scheme using attitude information of star tracker.

3.3 Kalman filter formulation

(1) SDINS dynamic equations

Integration Kalman filter design requires a mathematical error model for INS. Among INS error models, the following psi angle error model is selected as in Eqs. (4)–(6).

\[ \delta R^v = -\dot{\theta}^v_{\theta \psi} \times \delta R + \delta N^v \]  
\[ \delta N^v = \Omega^v \delta R^v - (\dot{\delta v}^n + \dot{\theta}^v_{\phi \psi} \times \delta v^v - \delta \psi^v \times \Omega^v + C^v_{\psi} \delta \psi^b) \]  
\[ \delta v^m = -\dot{\theta}^m_{\phi \psi} \times \delta v^m - C^m_{\psi} \delta \psi^b \]

where \( \delta R^v, \delta N^v \) and \( \delta v^m \) are the position, velocity and attitude error vectors; \( \dot{\theta}^v_{\theta \psi}, \dot{\theta}^v_{\phi \psi} \) and \( C^v_{\psi} \) are the estimated angular rate of the navigation frame relative to the earth frame in navigation frame, the estimated angular rate of the navigation frame relative to the inertial frame in the
navigation frame and the estimated direction cosine matrix from body frame to navigation frame; $\delta \Omega^b$ and $\delta \omega^b$ are the accelerometer bias and the gyroscope bias in body frame. The $\Omega_x$ is diagonal matrix given by

$$\Omega_x = \begin{bmatrix} \omega_x & 0 & 0 \\ 0 & \omega_y & 0 \\ 0 & 0 & 2\omega_z \end{bmatrix}$$ (7)

where $\omega_z$ is the Schuler frequency.

In state-space representation, Eqs. (4)–(6) can be expressed as follows

$$x(t) = F(t)x(t) + w(t)$$ (8)

$$F = \begin{bmatrix} F_i & F_f \end{bmatrix} \begin{bmatrix} x_i \\ x_f \end{bmatrix} + \begin{bmatrix} w_i \\ 0 \end{bmatrix}$$ (9)

Error state variable $x$ is represented as follows

$$x_i = \begin{bmatrix} \delta \Phi^i \delta h^i \delta V^N \delta V^E \delta \Psi^N \delta \Psi^E \end{bmatrix}$$ (9)

$$x_f = \begin{bmatrix} \delta \Phi^f \delta \Phi^f \delta \Phi^f \delta \Phi^f \delta \Psi^N \delta \Psi^E \delta \Psi^N \delta \Psi^E \end{bmatrix}$$ (10)

Process noise $w$ is

$$w = \begin{bmatrix} w_i \\ w_i \\ w_i \\ w_i \\ w_i \end{bmatrix}$$ (11)

In Eq. (8) $F_i$ and $F_f$ are represented as Eq. (12) and Eq. (13).

(2) Measurement model using position information of the star tracker

The difference between the position $(\Phi, \lambda, h)$ of star sensor and the position $(\Phi_{INS}, \lambda_{INS}, h_{INS})$ of SDINS can be represented as Eq. (14).

$F_f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & C^p & 0 \\ 0 & 0 & C^m \end{bmatrix}$ (12)

$$F_i = \begin{bmatrix} \frac{V_i \cos \Phi}{R \cos \Phi} & 0 & 0 \\ 0 & \frac{V_i \sin \Phi}{R \cos \Phi} & \frac{1}{R} \\ -2V_i \cos \Phi & 0 & -1 \end{bmatrix}$$ (13)

(3) Measurement model using position information of the star tracker

The relation between the attitude $C^p$ of star sensor and attitude $C^m$ of SDINS is as follows

$$z_p = \begin{bmatrix} \Phi - \Phi_{INS} \\ \lambda - \lambda_{INS} \\ h - h_{INS} \end{bmatrix} = \begin{bmatrix} \delta \Phi \\ \delta \lambda \\ \delta h \end{bmatrix} + v$$ (14)

$$Z_s = C^p C^m C^p - I$$ (15)

$$P = \Delta C^p C^m = \begin{bmatrix} \frac{-\delta \Phi \cos \Phi \cos \lambda - \delta \Phi \sin \lambda}{\delta \Phi \sin \lambda} & \frac{-\delta \Phi \sin \lambda}{\delta \Phi \cos \lambda} & \frac{-\delta \Phi \sin \lambda}{\delta \Phi \sin \lambda} \\ \frac{-\delta \Phi \sin \phi}{\delta \Phi \cos \lambda} & 0 & \frac{\delta \Phi \sin \lambda}{\delta \Phi \cos \lambda} \\ \frac{\delta \Phi \cos \lambda + \delta \Phi \sin \lambda}{\delta \Phi \cos \lambda} & \frac{\delta \Phi \sin \lambda}{\delta \Phi \cos \lambda} & \frac{-\delta \Phi \sin \lambda}{\delta \Phi \sin \lambda} \end{bmatrix}$$ (17)

The element, $[-(2,3) (1,3) -((1,2)]^T$ of P is

$$= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$ (18)
The third term of Eq. (16) can be expressed as

\[
C_{v}'M_{v} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\begin{bmatrix}
0 & -\mu_z & \mu_y \\
\mu_z & 0 & -\mu_x \\
-\mu_y & \mu_x & 0
\end{bmatrix}
\begin{bmatrix}
C_{11} & C_{21} & C_{31} \\
C_{12} & C_{22} & C_{32} \\
C_{13} & C_{23} & C_{33}
\end{bmatrix}
\]

(19)

The element, \([-2(3), (1,3) - 1(2,3)]\) of Eq. (19) is

\[
\begin{bmatrix}
C_{12}C_{33} - C_{23}C_{32} \\
C_{23}C_{31} - C_{21}C_{33} \\
C_{13}C_{22} - C_{12}C_{23}
\end{bmatrix}
\begin{bmatrix}
0 \\
\mu_z \\
\mu_y
\end{bmatrix}

(20)

Finally, kalman filter measurement equation can be derived as follows.

\[
Z_k = H_k \begin{bmatrix}
\Phi_k \\
\Psi_k \\
\Theta_k
\end{bmatrix}
\begin{bmatrix}
\mu_x \\
\mu_y \\
\mu_z
\end{bmatrix}

(21)

where,

\[
H_k = \begin{bmatrix}
0 \\
-C_1 \cos \phi \cos \lambda - C_2 \cos \phi \sin \lambda - C_3 \sin \phi \\
C_1 \cos \phi \sin \lambda + C_2 \cos \phi \cos \lambda - C_3 \cos \phi \sin \lambda
\end{bmatrix}
\]

4. SIMULATION RESULTS

For the validation of the proposed algorithms, a computer simulation is carried out to analyze effects of the method for stellar-inertial navigation system. The error elements of IMU(Inertial Measurement Unit) is specified as in Table 1. The vehicle trajectory profile used is displayed in Figure 5. The total maneuver time is 2000 sec and the velocity increases at 170 m/sec within the period from 180 sec to 200 sec.

<table>
<thead>
<tr>
<th>Table 1 Specification of sensor error elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error sources</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>initial position error</td>
</tr>
<tr>
<td>initial velocity error</td>
</tr>
<tr>
<td>initial horizontal attitude error</td>
</tr>
<tr>
<td>initial vertical attitude error</td>
</tr>
<tr>
<td>accelerometer bias</td>
</tr>
<tr>
<td>accelerometer noise</td>
</tr>
<tr>
<td>gyro bias</td>
</tr>
<tr>
<td>gyro noise</td>
</tr>
<tr>
<td>scale factor error</td>
</tr>
</tbody>
</table>

Table 2 Characteristics of a star sensor

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Of View</td>
<td>8' × 8'</td>
</tr>
<tr>
<td>Sensitivity Range(Mv)</td>
<td>+2 to +6</td>
</tr>
<tr>
<td>Accuracy(arcsec, 1σ) in Roll and Pitch</td>
<td>6</td>
</tr>
<tr>
<td>Update Rate(Hz)</td>
<td>10</td>
</tr>
<tr>
<td>Acquisition Time</td>
<td>6</td>
</tr>
<tr>
<td>(Full Field, second)</td>
<td></td>
</tr>
<tr>
<td>Maximum Number of Stars Tracked</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 6 and 7 are the Strap-Down Inertial Navigation (SDIN) simulation results using only INS without compensation using the star tracker. The position error is over about 150 m at 130 sec. The position error and attitude error diverge in process of time.
Figure 8 and 9 show the compensated INS simulation results using the measurement equation of kalman filter with the derived position information from chapter 3.3.

In compensating the navigation error of INS using the derived position information from celestial navigation, the position error is compensated effectively. But attitude error increases gradually.

Figure 11 and 12 show the compensated INS simulation results using attitude information from star tracker. Kalman filter measurement equation is derived using attitude information from chapter 3.3.

The position error and attitude error are bounded under 1500m and 0.16mrad.

Figure 14 and 15, given maneuvering of about 15 degree at about 500 sec under the same simulation condition of Figs. 11~12, show the reduction of position error and attitude error as compared with Figs. 11~12. In conclusion, Observability is the important factor at scheme using attitude information and is improved through attitude maneuvering.
5. SUMMARY AND CONCLUSION

In this paper, we suggest scheme using attitude and position information calculated by the celestial navigation algorithm. For the validation of the proposed algorithms, a computer simulation is carried out to analyze effects of the method for stellar-inertial navigation system. Finally, by interpreting computer simulation results, the suggested approach turns out to be effective to improve navigation errors which are compared with a case without the stellar augmentation. The simulation results of proposed scheme using position information show that position error is under 250m for a long time (about 1 hour). The other simulation results of scheme using attitude information show that position error and attitude error is under 1.5km and 0.15mrad for a long time.

Therefore, these techniques provide new schemes for SDINS navigation system using star tracker.

REFERENCES