

Study of the Robustness Bounds with Lyapunov-Based Stability Concept

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Abstract

The purpose of this project is the derivation and development of techniques for the new estimation of robustness for the systems having uncertainties. The basic ideas to analyze the system which is the originally nonlinear is Lyapunov direct theorems. The nonlinear systems have various forms of terms inside the system equations and this investigation is confined in the form of bounded uncertainties. Bounded means the uncertainties are with same positive/negative range. The number of uncertainties will be the degree of freedoms in the calculation of the stability region. This is so called the robustness bounds. This proposition adopts the theoretical analysis of the Lyapunov direct methods, that is, the sign properties of the Lyapunov function derivative integrated along finite intervals of time, in place of the original method of the sign properties of the time derivative of the Lyapunov function itself. This is the new sufficient criteria to relax the stability condition and is used to generate techniques for the robust design of control systems with structured perturbations. Using this relaxing stability conditions, the selection of Lyapunov candidate function is of various forms. In this paper, the quadratic form is selected. This generated techniques has been demonstrated by recent research interest in the area of robust control design and confirms that estimation of robustness bounds will be improved upon those obtained by results of the original Lyapunov method. In this paper, the symbolic algebraic procedures are utilized and the calculating errors are reduced in the numerical procedures. The application of numerical procedures can prove the improvements in estimations of robustness for one- and more structured perturbations. The applicable systems is assumed to be linear with time-varying with nonlinear bounded perturbations. This new techniques will be extended to other nonlinear systems with various forms of uncertainties, especially in the nonlinear case of the unstructured perturbations and also with various control method.

I. Introduction

The Lyapunov direct method has been used in the various fields of applications for control system analysis and design, especially for the determination of the stability region. This is the most general method for the estimation of the stability of nonlinear and time-varying systems. This method was applied to estimation of the domain of attraction of autonomous

nonlinear systems by Vannelli and Vidyasagar [13]. Brayton and Tong [5] introduced computer-generated Lyapunov functions. In this paper, uncertainties are the compact sets in which they may range. Based upon this knowledge, state feedback controls have been designed to assure uniform asymptotic stability of the zero state for all possible system responses. Barmish and Leitmann [2] completed research for uncertainty threshold estimation. According to Patel and Toda [10], the bounds of perturbation can be computed numerically, providing a useful quantitative measure of robustness for asymptotically stable systems. The Lyapunov method to testing for the stability of state-space models was studied by Yedavalli and Liang [15] and the principal topic was the reduction of the conservatism of the stability robustness bounds. Application of the Lyapunov approach has also been used for the estimation of controller robustness, thus developing new technique in the parameter space approach to control systems and using alternatives to the conventional methods of Routh-Hurwitz and Nyquist. This is the first step in the robust design of control systems subject to structured perturbations has served to shift research activity toward parameter space methods and enlargement of the scope of the approach to include the Lyapunov method as well as frequency domain concept [5]. The objective of this investigation is the development of a new Lyapunov technique modified from original concept. The proposed technique provides improved results for two basic robust control design problems, the robust stability problem and the uncertain system stabilization problem. Also this approach involves application of the Lyapunov direct method to control design for time-variant, nonlinear systems with bounded perturbations.

II. Theoretical Analysis

1. Study of the stability conditions

In 1982, A.M. Lyapunov presented his first and second method for determining the stability of dynamic systems as described by ordinary differential equations. The first method considered entirely of procedures in which the explicit form of the solutions of the differential equations were used for the analysis, a method which has proved to be fundamental to the stability analysis of equilibrium points. But this method is applicable only to local stability at the point of interest. The second method does not require the solution of differential equations. Thus, the Lyapunov's second method is suitable to the stability

analysis of nonlinear systems for which exact solutions may not be obtained. The Lyapunov's direct method (other name of second method) provides improved stability evaluation and is powerful (convenient) method by the use of the energy function concept. It is accepted that a vibratory system is asymptotically stable if its total energy (a positive definite function) is continually decreasing (that is, the time derivative of the total energy must be negative definite) until reaching a state of equilibrium. The Lyapunov direct method is based upon the fact that if the system has an asymptotically stable equilibrium state, then the stored energy of the system which is disturbed within the domain of attraction decays with increasing time. But there is no simple technique to find an energy function. In Lyapunov's direct method, the sign property of $V(x, t)$ and that of its time derivative $\dot{V}(x, t)$ provide information on the stability or instability of an equilibrium state without getting the analytic solution.

Theorem 1. Suppose that a system is described by $\dot{x} = f(x, t)$ where $f(0, t) = 0$ for all t . The system is continuous when solutions exist. If there exists a scalar function $V(x, t)$, having continuous, first partial derivatives and satisfying the following conditions,

- (1) $V(x, t)$ is locally positive definite and
- (2) $\dot{V}(x, t) \leq 0, t \geq t_0, x \in B_r,$

then the equilibrium state at the origin is stable. And the equilibrium point at time t_0 is uniformly asymptotically stable over the interval $[t_0, \infty)$ if there exists a continuously differentiable decrescent locally positive definite function V such that $-\dot{V}$ is an locally positive definite function.

Theorem 2. Consider the system described by $\dot{x} = Ax$, where x is a state vector ($n \times n$ dimensional vector) and A is an $n \times n$ constant nonsingular matrix. It is a necessary and sufficient condition that the equilibrium state $x = 0$ be asymptotically stable if, given any positive definite matrix Q , there exists a positive definite matrix P such that $A^T P + FA = -Q$. The scalar function $x^T P x$ is a Lyapunov function for this system. To determine the elements of the P matrix (positive definite matrix), the matrices $A^T P + FA$ and $-Q$ are equated element by element. And it is convenient to select $Q = I$, the elements of P are determined and tested for positive definiteness. The definition of the robustness of control systems is composed of two ingredients:

(i) a system property (e.g., a stability margin in the frequency domain), and

(ii) a class of perturbations against which the system properties are robust (e.g., uncertain physical parameters, neglected actuator dynamics and non-linearity, modeling uncertainty, non-ideal controller implementation, sensor or actuator failure.)

Stability conditions vary with different system's parameter conditions. However, parameter with certain conditions stated in the system equations can be determined if the system continues to be stable over a large parametric space. In this proposition, the techniques can improve the parametric range of the structured perturbations considered to be the robustness bounds.

2. New concept of stability and robustness

The objective of the current investigation is to improve the robustness estimates of dynamic systems with structured uncertainties, using Lyapunov stability conditions to weaken the stability conditions formulated in conventional Lyapunov theorems. The main concept of this propose is the adoptions of the sign properties of the Lyapunov function derivative integrated along the finite interval of time rather than the sign properties of the derivative itself, which has been the traditional method of judging the system stability.

Consider a system

$$\dot{x} = f(t, x), f(t, 0) = 0, \quad (1)$$

where $x \in R^n, \|x\| < H$. Let $V = V(t, x)$ be a continuously differentiable function, such that it is locally positive-definite on some ball, B_r , centered at

O . Then, let $\dot{V}(t, x)$ denote a derivative of V along the system given in (1) and let $x(t, t_0, x_0)$ exist for $t \in [t_0, t_0 + T]$.

Then, consider the expression

$$\int_{t_0}^t V(\tau, x(\tau, t_0, x_0)) d\tau \quad (2)$$

The continuity of $V(t, x)$ and the fact $V(t, 0) = 0$ are considered and the following conclusion can be obtained.

Conclusion 1. For any constant $\varepsilon_3 > 0$ and for any interval $[t_0, t_0 + T]$, there exists a constant $\delta_3 > 0$, such that if $\|x_0\| < \delta_3$, then

$\left| \int_{t_0}^{t_0+u} V(\tau, x(\tau, t_0, x_0)) d\tau \right| < \varepsilon_3$ for $u \in [t_0, t_0 + T]$; when the above integral on the interval $[t_0, t]$ is estimated, a group property of the solution to (1) is used. Assume that x_0 is so chosen that the solution exists and remains within the ball, B_r , on the interval $[t_0, t]$. Let the sequence of time instants $\{\tau_i\}$, $i = 0, 1, \dots, m$, $\tau_0 = t_0$, $\tau_m = t$, $\tau_{i+1} < \tau_i$ divide the interval $[t_0, t]$ on

m sub-intervals

$$[\tau_{i+1}, \tau_i], \quad i=0, 1, \dots, m-1.$$

Then, write

$$\int_{t_0}^t V(\tau, x(\tau, t_0, x_0)) d\tau = \sum_{i=0}^{m-1} \int_{\tau_{i+1}}^{\tau_i} V(\tau, x(\tau, t_0, x_0)) d\tau$$

$$(3) \quad \text{and} \quad x(\tau, t_0, x_0) = x(\tau, \tau_i, x_i) \quad (4)$$

Relation (4) allows the transformation of (3) to the following form

$$\int_{t_0}^t V(\tau, x(\tau, t_0, x_0)) d\tau = \sum_{i=0}^{m-1} \int_{\tau_{i+1}}^{\tau_i} V(\tau, x(\tau, \tau_i, x_i)) d\tau.$$

(5) Thus, it is possible to formulate the following theorem:

Theorem 3. In the system given (1), if there exist

- (i) a continuously differentiable and locally positive-definite function $V(t, x)$ and
- (ii) a bounded function $T(t, x)$ defined for $t \in [0, \infty]$, $x \in B_r$, and having a positive lower bound such that the function

$$V^*(x) = \int_{t-T(t,x)}^t V(\tau, x(\tau, t, x)) d\tau \quad (6)$$

exists and fulfills the condition $V^*(t, x) \leq 0$, for $(t, x) \in [0, \infty) \times B_r$ (7)

then the trivial solution of (1) is Lyapunov stable.

<Proof> The proof differs from the conventional proof of the Lyapunov theorem. As is known, it is sufficient to prove stability for a selected initial instants t_0 , and $t_0=0$ is selected. By virtue of the definition of $V(t, x)$, there exists a continuous and positive-definite function $W(x)$ such that

$$V(t, x) \geq W(x) > 0, \quad \text{for } x \neq 0, \quad x \in B_r, \\ V(t, 0) = W(0) = 0.$$

(8)

Selected the sphere $S_\varepsilon = \{ \|x\| = \varepsilon \}$ such that $S \in B_r$. Using the fact that the function $V(0, x)$ is continuous and $V(0, 0) = 0$, it may be concluded that there exists a neighborhood $\|x\| < \delta_2 < \varepsilon$, such that for every $\|x_0\| < \delta_2$, $0 \leq V(0, x_0) < \frac{\alpha}{2}$.

By the conclusion 1, δ_3 is chosen so that for every x_0 satisfying $\|x_0\| < \delta_3$,

$$\left| \int_0^u V(\tau, x(\tau, 0, x_0)) d\tau \right| < \frac{\alpha}{2}$$

(9)

for $u \in [0, T]$. Then, denoting that $\delta_4 = \min(\delta_2, \delta_3)$ for $\|x_0\| < \delta_4$,

$$0 \leq V(0, x_0) < \frac{\alpha}{2} \quad (10)$$

and

$$\left| \int_0^u V(\tau, x(\tau, 0, x_0)) d\tau \right| < \frac{\alpha}{2}$$

(11)

for $u \in [0, T]$. Thus, consider an arbitrary non-trivial solution with initial condition x_0 , such that $\|x_0\| < \delta_4$, given that trajectory of this solution remains entirely inside of the sphere S_ε , that is,

$$\|x(t, 0, x_0)\| < \varepsilon, \quad \text{for } t \in [0, \infty) \quad (12)$$

Assume the contrary, at some instant $t=t^*$, the point of trajectory is for the first time hit the boundary of the sphere S_ε :

$$\|x(t, 0, x_0)\| < \varepsilon \quad \text{for } t \in [0, t^*) \\ \|x(t, 0, x_0)\| = \varepsilon \quad (13)$$

The value of $V(t, x)$ is obtained by

$$V(t^*, x(t^*, 0, x_0)) = V(0, x_0) + \int_0^{t^*} V(\tau, x(\tau, 0, x_0)) d\tau \quad (14)$$

Denote $x(t^*, 0, x_0) = x^*$, $t^* = \tau_0$, then the sequence of instants, $\{\tau_i\}$, $\tau_{i+1} < \tau_i$, and the sequence of points, $x_i \in B_r$, defining

$$\tau_1 = \tau_0 - T(\tau_0, x^*), \quad x_1 = x(\tau_1, 0, x_0), \\ \tau_{i+1} = \tau_i - T(\tau_i, x_i), \quad x_{i+1} = x(\tau_{i+1}, 0, x_0). \quad (15)$$

The integral of the right-hand side of (14) is transformed, writing

$$\int_0^{t^*} V(\tau, x(\tau, 0, x_0)) d\tau = \sum_{i=0}^{m-1} \int_{\tau_{i+1}}^{\tau_i} V(\tau, x(\tau, \tau_i, x_i)) d\tau \quad (16)$$

From assumption (7), all of the above integration, with the exception of the last time step integration of the integral (16), are less than or equal to zero. Using (14), it may be estimated

$$V(t^*, x(t^*, 0, x_0)) \leq V(0, x_0) + \left| \int_0^{\tau_{m-1}} V(\tau, x(\tau, 0, x_0)) d\tau \right| \quad (17)$$

The evaluation from the (11) and (12) will be

$$V(t^*, x(t^*, 0, x_0)) < \alpha \quad (18)$$

However, $x(t^*, 0, x_0)$ always remains inside of the ball S_ε , by virtue of the condition (8)

$$V(t^*, x(t^*, 0, x_0)) \geq \alpha. \quad (19)$$

This fact suggests that the accident of the first hit on the boundary surface S_ε will not happen under these circumstances. This theorem was proven by the contradiction concept.

Other related theorems for under several conditions of the trivial solution to the system given in (1) are given:

Theorem 4. Consider a system (1), as defined in Theorem 3. If there exist:

(i) a continuously differentiable, locally positive-definite, and decrescent function $V(t, x)$,

(ii) a bounded function $T(t, x)$ defined for $t \in [0, \infty]$, $x \in B_r$, and having a positive lower bound, and

(iii) a continuous, locally positive-definite function $W(x)$, such that the function

$$-V^*(t, x) = -\int_{t-T(t, x)}^t V(\tau, x(\tau, t, x)) d\tau \quad \text{fulfills}$$

the conditions

$$-V^*(t, x) \geq W(x) \quad , \quad \text{for } x \in B_r \quad \text{and}$$

$-V^*(t, 0) = W(0)$, then the trivial solution of (1) is asymptotically stable. In this theorem, it is not

required that $V^*(t, x)$ is a continuous function. As may be seen from Theorem 3, Theorem 4. is the stronger of the two. Since this is the case, it is only necessary to prove that $\lim_{t \rightarrow \infty} \|x(t, 0, x_0)\| = 0$.

Theorem 5. Consider a system (1), as defined in Theorem 2, and let all solutions of (1) be defined in the future. If there exist :

(i) a continuously differentiable, positive definite, and decrescent function $V(t, x)$,

(ii) a bounded function $T(t, x)$ defined for $t \in [0, \infty]$, $x \in B_r$, and having a positive lower bound, and

(iii) a continuous, locally positive-definite function $W(x)$, such that the function (5)

$$V^*(t, x) = \int_{t-T(t, x)}^t V(\tau, x(\tau, t, x)) d\tau \quad \text{fulfills the}$$

conditions $-V^*(t, x) \geq W(x)$, for $x \in B_r$, and

$-V^*(t, 0) = W(0)$, then the trivial solution of (1) is globally asymptotically stable. The fact that the

solution of (1) are defined enables estimation of the upper bound of the integral $\int_0^u V(\tau, x(\tau, 0, x_0)) d\tau$

for $u \in [0, T)$. The following corollary provides global asymptotic stability conditions for the trivial solution of the autonomous system for the case when $V=V(x)$:

Corollary 1. Consider a system

$$\dot{x} = f(x) \quad , \quad f(0) = 0 \quad (20)$$

where $x \in R^n$. If there exist :

(i) a continuously differentiable, positive definite function $V(x)$,

(ii) a bounded function $T(x)$ defined for $x \in R^n$, and having a positive lower bound, and

(iii) a continuous, positive definite function $W(x)$, such that the function

$$V^*(x) = \int_{-T(x)}^0 V(x(\tau, 0, x)) d\tau \quad (21)$$

fulfills the conditions $-V^*(x) \geq W(x)$ and

$V^*(0) = W(0)$, then the trivial solution of (20) is globally asymptotically stable. The procedure based on Corollary 1 is then a natural extension of the Lyapunov direct method procedure.

The new concepts and approach techniques to dynamic systems have been introduced in this section.

III. Main Results

From the theoretical analysis in the previous section, actual dynamical systems are approached by application of the Lyapunov direct method to improve the robustness bounds. In this study, systems with structured perturbations are introduced for the purpose of problem analysis with the proposed numerical and computational procedure. Theorem 5 is applied to the investigation of the robustness of linear systems with structured uncertainties. The Lyapunov candidate function which has the quadratic form was selected at first. The bounds of uncertainty were defined by calculating the Lyapunov derivatives. Then, new and higher bounds of uncertainty was selected, with the function $V^*(x)$ considered for the initial conditions belonging to a unit sphere. For these initial conditions, for which the Lyapunov derivative remains negative, there is no need to determine a solution. For the remaining solutions, integration is extended until the function \dot{V}^* is negative. The robust design of control systems subject to structured perturbations is a natural application of the results which are presented.

Example 1. Consider the system

$$\dot{x} = Ax + g(t, x)Gx \quad , \quad x \in R^2 \quad \text{where } g(t, x) \text{ is a scalar function, } A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad , \quad G = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad ,$$

and the bound k on $g(t, x)$ is such that if $|g(t, x)| < k$, the trivial solution of the system equation is asymptotically stable. A quadratic form of the Lyapunov function V is chosen as Lyapunov candidate function $V = x^T F x$,

$V = x^T (A^T P + F A) x$. Solving for the matrix P to obtain the best Lyapunov function results in

$$P = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad \text{and } V = x_1^2 + x_1 x_2 + x_2^2 \quad . \quad \text{Then,}$$

V is differentiated with respect to time,

$$\frac{dV}{dt} = 2x_1 \dot{x}_1 + \dot{x}_1 x_2 + x_1 \dot{x}_2 + 2x_2 \dot{x}_2 \quad , \quad \text{and } \dot{x}_1 \text{ and}$$

\dot{x}_2 in the system equation are replaced:

$$\begin{aligned} \frac{dV}{dt} &= 2x_1 x_2 + x_2^2 + x_1 [(-1 + g)x_1 - x_2] \\ &\quad + 2x_2 [(-1 + g)x_1 - x_2] \\ &= -[(1 - g)x_1^2 + (1 - 2g)x_1 x_2 + x_2^2] < 0 \end{aligned}$$

If V fulfills the condition $V < 0$, then the above equation must fulfill the following conditions:

(1) $1 - g > 0$, $g < 1$, and

$$(2) (1-2g)^2 - 4(1-g) < 0, |g| < \frac{\sqrt{3}}{2} = 0.8660.$$

If the case is positive, then the system is asymptotic stable. The numerical procedure is based on the result $|g|=0.96$, signifying a 10.8% extension in the estimate of robustness. To consider the unit sphere in two-dimensional space, first select k with the same extent of estimate, searching the areas that do not satisfy $V < 0$. Corollary 1 is then used to check the

$$\text{values of } V^*(x) = \int_{-T(x)}^0 V(x(\tau, 0, x)) d\tau \text{ and}$$

$$\bar{V}(t, x) = V(t, x) - V(t, T(t, x), x(t - T(t, x), t, x)) \quad (22)$$

If the value of (22) is less than zero, this initial state is stable. In the mathematical point of view, differential equations are in ideal terms solved about the state variable. For the area which does not satisfy the condition $V < 0$, integrate backward in time, selecting the time T ,

$$x(-T) = e^{-AT}x(0) + \int_0^T e^{A(\tau-T)}g(t, x)Gx d\tau$$

the exact solution from the system equation, which may then be used to check the value of V , the proposed technique.

Example 2. Consider the three dimensional case proposed by Siljak[12], originally introduced by Zhou and Khargonekar [17]. The linear constant system equation

$$S: \dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} u \quad \text{and}$$

$$y = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x \text{ is driven by the output feedback}$$

$$u = - \begin{bmatrix} 1-k_1 & 0 \\ 0 & 1-k_2 \end{bmatrix} y.$$

The final closed loop system is :

$$\hat{S}: \dot{x} = \begin{bmatrix} -2+k_1 & 0 & -1+k_1 \\ 0 & -3+k_2 & 0 \\ -1+k_1 & -1+k_2 & -4+k_1 \end{bmatrix} x.$$

This case is a two-degree of freedom dynamic system with two structured perturbation, k_1 and k_2 .

Applying the Lyapunov direct method, $V = x^T F x$ and

$V = x^T (A^T P + F A) x$ and the equation $A^T P + F A = -I$ is solved. It follows that

$$P = \begin{bmatrix} 2 & 9 & -1 \\ 7 & 476 & -14 \\ 9 & 83 & -11 \\ 476 & 476 & 476 \\ -1 & -11 & 1 \\ 14 & 476 & 7 \end{bmatrix}, \text{ where } P \text{ is the}$$

exact solution and is positive-definite. Then, V is calculated by the matrix :

$$A^T P + F A = \begin{bmatrix} \frac{3k_1-7}{7} & \frac{-25k_2+2k_1}{476} & \frac{2k_1}{7} \\ -\frac{25k_2+2k_1}{476} & \frac{36k_2-119}{119} & \frac{57k_2-2k_1}{476} \\ \frac{2k_1}{7} & \frac{-57k_2-2k_1}{476} & \frac{k_1-7}{7} \end{bmatrix}$$

If the matrix $A^T P + F A$ had been negative-definite, the selected system would always be asymptotically stable. The regions k_1 and k_2 should be $|k_1| < 1.60$ and $|k_2| < 2.74$ for $A^T P + F A$ to be negative-definite. To find the regions for k_1 and k_2 , the inequality equations are derived in a manner such that the negativeness of the matrix $A^T P + F A$ observes the Routh-Hurwitz criterion. After using the same proposed technique, the k_1 and k_2 regions can be extended. For $|k_1| < 1.60$, the region of k_2 is extended 2.97 which is an 8.3% improvement in robustness estimate.

IV. Conclusions

A new concept and numerical procedure to estimate the robustness of multi-dimensional systems with bounded perturbations has been presented in this investigation. New stability criteria and conditions were introduced and proven for selected examples. The sign properties of the Lyapunov function derivative integrated along finite intervals of time were considered, rather than upon the sign properties of the derivative itself. Theorems were formulated to serve as a basis for both analytical and numerical procedures. The results demonstrated improvements of the bounds and global asymptotic stability for selected ranges of parameters. An example of a two dimensional system was investigated by Radziszewski, who obtained estimation results of $k=0.866$. It was apparent that allowing $g(t, x) = -1$ results in an unstable trivial solution for a two dimensional system. The application of the proposed method of improving robustness bounds results in a reduction in the margin of the bound estimation (i.e., the difference between the exact bound and the estimated bound). The numerical procedure based on the presented results provides $k=0.96$ which is only 4% less than the maximum possible result and is 10% better than $k=0.866$.

For the three-dimensional system, originally considered by Zhou and Khargonekar[17], the results of the proposed technique when compared with results from previous research for the bounds for robust stability were as follows:

(a) Patel and Toda[11] : $|k_i| < 0.5207$

(b) Yedavalli and Liang[15] : $|k_i| < 0.81577$

(c) Zhou and Khargonekar[17] : give three different conditions, any one of which is sufficient : (i) $k_1^2 + k_2^2 < 1$, (ii) $0.6052|k_1| + 0.3512|k_2| < 1$, (iii) $|k_i| < 1.5533$.

The results for the proposed method are $|k_1| < 1.60$ and $|k_2| < 2.97$. As may be seen the proposed method allows for improvements in each parameters region. New conditions and theorems are utilized and improved parametric robustness bounds are derived. Further research of robustness bounds based upon Lyapunov approach should be directed toward the study of various practical systems and new generations of Lyapunov function.

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