D* Model Matching Control System for Four Wheel Steering

Naoki ASARA*, Yasuhiro OSA, Shigeru UCHIKADO*** and Kimio KANAI******

* The Advanced Course of Mechanical Systems Engineering, Kobe City College of Technology, Kobe, Japan
** Department of Mechanical Engineering, Kobe City College of Technology, Kobe, Japan
(Tel: +81-78-795-3210; E-mail: osa@kobe-kosen.ac.jp)
*** School of Science and Engineering, Tokyo Denki University, Saitama, Japan
(Tel: +81-492-96-2911; E-mail: uchikado@j.dendai.ac.jp)
**** NISSAN Motor Company, Atsugi Technical Center, Atsugi, Japan
(Tel: +81-46-270-1844; E-mail: k-kimio@muh.biglobene.jp)

Abstract: D* criterion is defined as a reference of the handling quality and ride comfortableness for lateral-directional automobile motion. However it is generally difficult to obtain the satisfied handling quality and ride comfortableness based on D* criterion by conventional two wheel steering system. In this study, a design method of model matching control system is proposed to obtain the satisfied D* response of 4 Wheel Steering.

Key Words: D* criterion, 4 Wheel Steering, Model Matching, Control

1. INTRODUCTION

Four Wheel Steering (4WS) [1] is a system which steers the rear wheels by any control method related with the angle of front wheels steered by the driver. It has been developed to improve the mobility and the stability of conventional 2WS. 4WS provides many advantages, for example, it can make the turning radius small for parallel parking at low speed and realize the stable lane change [2] and avoidance against obstacles at high speed. Moreover 4WS may have the possibility to improve the ride comfortableness and handling quality, but there is not general reference for them and very little has been researched for them.

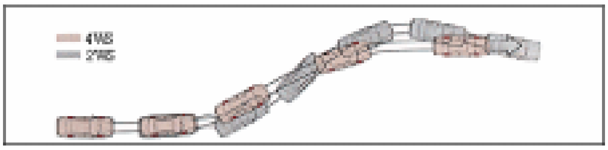

Fig.1 Advantage of 4WS

At present time as a reference for the handling quality and ride comfortableness on longitudinal motion of aircraft, C* criterion [3] is used. It is defined as a scalar expressed with the linear sum of vertical acceleration at pilot position and pitch rate. Likewise as a reference for the handling quality and ride comfortableness on lateral-directional motion of automobile, D* criterion [4] is proposed. This criterion is an application of above C* criterion and defined as the following equation.

$$D^*(t) = \{ d\,\dot{v}(t) + (1-d)V\omega(t) \}/g \qquad (1)$$

Where $\mathbf{v}(t)$: lateral-longitudinal velocity[m/s], V: forward speed[m/s], $\boldsymbol{\omega}(t)$: yaw rate[rad/s], d: arbitrary weight constant ($0 \le d \le 1$) and g: gravity acceleration[9.8m/s^2].

To control above D*(t) as output, some studies [4] using an optimum control law and others have been presented. Yet the problems of selection of the optimum weight constant d and so on are remained.

Also for an automobile on a running condition, considering the forward wheel angle as the reference model input, we had tried to construct the SISO model matching system which controls the D* response by using the rear wheel angle as the control input, but the satisfied response could not be obtained for any value of weight constant d, because the inverse system becomes unstable or almost critical stable in spite of any value of d.

In this study adopting the other point of view for above D*(t), an MIMO model matching [5-6] control system for 4WS is constructed using $\dot{\mathbf{v}}(t)/g$ and $V\boldsymbol{\omega}(t)/g$ in Eq.(1) as the outputs and the forward/rear wheel angles as the control inputs. The control law is determined simply by our proposed linear model matching method. Each output is controlled to match each reference model provided by driver, as the result D* response which is the sum of each output converges within the range of some envelope. Moreover depending on the values of reference model inputs, this control system has another merit which realize only the yaw rate control (running on the arbitrary turning radius) or only the lateral-directional acceleration control (crab-wise) [1,4]. This system may be able to give a hint for handling quality or ride comfortableness based on D* criterion.

2. Equation of 4WS motion

Fig.2 shows the 2 wheels model [7] which is equivalent to the model of 4WS turning motion.

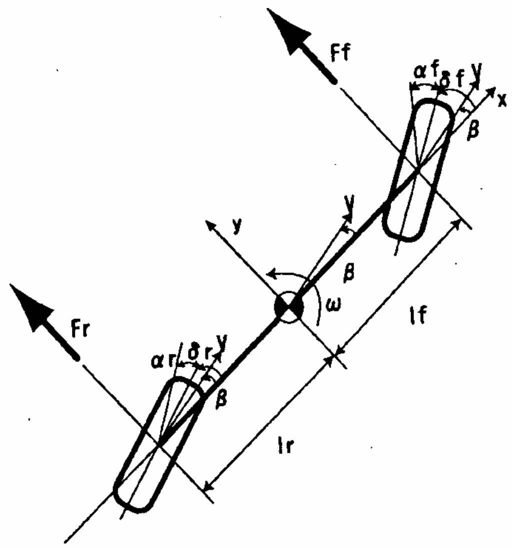

Fig.2 Model of 4WS turning motion

Where y(t): lateral-directional displacement[m], β(t): side slip angle[rad], $\delta_f(t)$&$\delta_r(t)$: steering angle[rad], $F_f(t)$&$F_r(t)$: cornering force[N], $\alpha_f(t)$&$\alpha_r(t)$: slip angle[rad], l_f&l_r: length between C.G. and wheel[m], and the subscript "f" means the front and the subscript "r" means the rear.

2.1 Equation of 4WS turning motion

From Fig.2, about the lateral direction and around the center of gravity the following equations can be constructed.

[Lateral direction]

$$M\{\dot{v}(t) + V\omega(t)\} = F_f(t) + F_r(t) \quad (2)$$

[Around the center of gravity]

$$I\dot{\omega}(t) = F_f(t)\, l_f + F_r(t)\, l_f \quad (3)$$

Where M: mass[kg], I: moment of inertia[kgm^2] and $v(t) = dy(t)/dt$. Also cornering forces of front and rear wheels, $F_f(t)$ and $F_r(t)$ in Eq. (2) and (3), are generated as Eq.(4) and (5) respectively.

[Cornering force of front wheel]

$$F_f(t) = C_f\, \alpha_f(t) \quad (4)$$

$$\alpha_f(t) = -[\{v(t) + l_f\, \omega(t)\}/V - \delta_f(t)]$$

[Cornering force of rear wheel]

$$F_r(t) = C_r\, \alpha_r(t) \quad (5)$$

$$\alpha_r(t) = -[\{v(t) - l_r\, \omega(t)\}/V - \delta_r(t)]$$

Where C_f and C_r: cornering stiffness [N/rad].

2.2 Derivation of state space equation

In this subsection, construct the state space equation with respect to v(t) and ω (t) based on Eq.(2) –Eq.(5).

At first by the relations between Eq. (2) and Eq.(4)-(5), the following equation can be obtained.

$$\begin{aligned}\dot{v}(t) = &-\{(C_f+C_r)/MV\}v(t) \\ &-\{(MV^2 + C_f l_f - C_r l_r)/MV\}\,\omega(t) \\ &+(C_f/M)\,\delta_f(t) + (C_r/M)\,\delta_r(t)\end{aligned} \quad (6)$$

Next by the relations between Eq. (3) and Eq. (4)-(5), the following equation can be obtained.

$$\begin{aligned}\dot{\omega}(t) = &-\{(C_f l_f - C_r l_r)/IV\}v(t) \\ &-\{(C_f l_f^2 + C_r l_r^2)/IV\}\,\omega(t) \\ &+(C_f l_f/I)\,\delta_f(t) - (C_r l_r/I)\,\delta_r(t)\end{aligned} \quad (7)$$

Then based on Eq (6) and (7), the following state space equation Eq. (8) can be obtained.

$$\dot{x}(t) = Ax(t) + B\delta(t) \quad (8)$$

Where $x(t) = [v(t), \omega(t)]^T$, $\delta(t) = [\delta_f(t), \delta_r(t)]^T$ and A, B $\in \mathbf{R}^{2x2}$. Each element in the matrices A and B is as follows:

$a_{11} = -(C_f+C_r)/MV$, $a_{12} = -(MV^2 + C_f l_f - C_r l_r)/MV$,

$a_{21} = -(C_f l_f - C_r l_r)/IV$, $a_{22} = -(C_f l_f^2 + C_r l_r^2)/IV$,

$b_{11} = C_f/M$, $b_{12} = C_r/M$,

$b_{21} = C_f l_f/I$ and $b_{22} = -C_r l_r/I$

3. THE OUTPUT VECTOR BASED ON D* CRITERION

3.1 C* criterion [3]

As a reference for handling quality and ride comfortableness on longitudinal motion of aircraft, C* criterion is used. It is defined as a scalar value normalized by dividing the linear sum of vertical acceleration at the pilot position and pitch rate around the center of gravity by gravity acceleration and expressed as the following equation.

$$C^*(t) = \{n_{zp}(t) + V_{co}q(t)\}/g$$

Where $n_{zp}(t)$: vertical acceleration at pilot position [m/s^2], V_{co}: cross over speed [m/s] and q(t) : pitch rate [rad/s].

Also it is required that this time response C*(t) for step input converges within the range of an envelope shown in Fig.3.

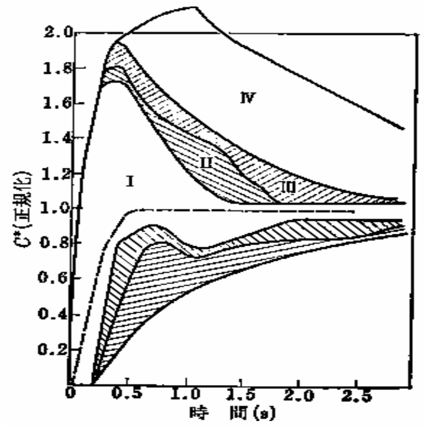

Fig.3 C* criterion

Where each range in Fig.3 is considered as follows:

Range I: Optimum response
Range II: Neither optimum nor danger
Range III: Case except Range I, II and IV
Range IV: Power approach response

Generally the better handling quality (speed of response) is derived, the worse ride comfortableness becomes. Also the personal differences exist for handling quality and ride comfortableness. Hence Range I is defined as the suitable range shows whether they are optimum or not. Usually longitudinal flight control systems are designed so as to make the step response converge within Range I.

3.2 D* criterion [4]

Likewise as a reference for the handling quality and ride comfortableness on lateral-directional motion of automobile, D* criterion is proposed. This criterion is an application of above C* criterion and defined as the following equation.

$$D^*(t) = \{d\dot{v}(t) + (1-d)V\omega(t)\}/g \quad (1)$$

The physical meaning of above D*(t) for 4WS is a scalar value normalized by dividing the sum of lateral-directional acceleration and centripetal acceleration occurred by yaw rate around the center of gravity by gravity acceleration.

3.3 The selection of output vector based on D* criterion

Based on Eq. (1), for an automobile on a running condition, considering the forward wheel angle as the reference model input, we had tried to construct the SISO model matching system which controls the D* response by using the rear wheel angle as the control input, but the satisfied response could not be obtained for any value of weight constant d, because the inverse system becomes unstable or almost critical stable in spite of any value of d. Then consider the following output vector.

Adopting the other point of view for above D*(t), consider $\dot{v}$ (t)/g and V ω (t)/g in the right hand of Eq. (1) as the two outputs, and define the following output vector.

$$\mathbf{y(t)=[y_1(t), y_2(t)]^T = [\dot{v}(t)/g, V\omega(t)/g]^T} \quad (9)$$

Hereby the inverse system does not become unstable for the same running condition, the lateral-directional acceleration and the centripetal acceleration occurred by yaw rate can be controlled directly.

4. FORMULATION OF THE PROBLEM

Consider the following discrete time linear state space equation and output equation as the controlled system [8].

[System Σ]

$$\mathbf{x(k+1) = A_D x(k) + B_D u(k)}$$
$$\mathbf{y(k) = Cx(k)} \quad (10)$$

Where $\mathbf{x(k)}\in R^n$, $\mathbf{y(k)}\in R^p$, $\mathbf{u(k)}\in R^p$ and $\mathbf{C}\in R^{pxn}$. Also $\mathbf{A_D}\in R^{nxn}$ and $\mathbf{B_D}\in R^{nxp}$ can be calculated as follows using $\mathbf{A}\in R^{nxn}$ and $\mathbf{B}\in R^{nxp}$ included in a general continuous time state space equation.

$$\mathbf{A_D} = e^{A\Delta T}, \quad \mathbf{B_D} = \int_0^{\Delta T} e^{A\tau} \mathbf{B} d\tau \quad (11)$$

On the other hand, consider the following discrete time system [8] as the reference model selected by the driver arbitrarily.

[System Σ $_M$: Reference Model]

$$\mathbf{x_M(k+1) = A_M x_M(k) + B_M u_M(k)}$$
$$\mathbf{y_M(k) = C_M x_M(k)} \quad (12)$$

Where $\mathbf{x_M(k)}\in R^m$, $\mathbf{y_M(k)}\in R^p$ and $\mathbf{u_M(k)}\in R^p$.

The objective of this study is to design a model matching control system which matches the output of controlled system y(k) in Eq.(10) to the output of reference model y_M(k) in Eq.(12).

5. SYNTHESIS OF DISCRETE TIME LINEAR MODEL MATCHING CONTROL SYSTEM

In this section the discrete time linear model matching [5-6] control law is derived.

For system **Σ** and system **Σ** $_M$ perform the following procedure.

[Step1] Consider the time-shift signals of the output y_1(k) and the reference model output y_{M1}(t), left-multiply them by time-shift operator “z”. Then using Eq.(10) and (12), the following equations can be obtained.

$$\mathbf{zy_1(k) = C_1A_Dx(k) + C_1B_Du(k)}$$
$$\mathbf{zy_{M1}(k) = C_{M1}A_Mx_M(k) + C_{M1}B_Mu_M(k)} \quad (13)$$

Next formally replace the above equations with as follows:

$$\mathbf{z^{f11}y_1(k) = Ca_{11}x(k) + Da_{11}u(k)}$$
$$\mathbf{z^{f11}y_{M1}(k) = Ca_{M11}x_M(k) + Da_{M11}u_M(k)} \quad (14)$$

Where $\mathbf{Ca_{11} = C_1A_D}$, $\mathbf{Ca_{M11} = C_{M1}A_M}$, $\mathbf{Da_{11} = C_1B_D}$ **and** $\mathbf{Da_{M11} = C_{M1}B_M}$. Also the left "1" of subscript "11" means the 1st output, the right "1" means the 1st power of z^1. In the above equations, when if $\mathbf{Da_{11}}$=/=**0**, replace the subscripts “11” with “1” and go to the next step. When if $\mathbf{Da_{11} = 0}$, repeat the time shift until the following equations can be obtained.

$$\mathbf{z^{f1j}y_1(k) = Ca_{1j}x(k) + Da_{1j}u(k)}$$
$$\mathbf{z^{f1j}y_{M1}(k) = Ca_{M1j}x_M(k) + Da_{M1j}u_M(k)} \quad (15)$$

Where $\mathbf{Da_{1j}}$=/=**0** and it is assumed that "j" which satisfies the above equations exists. Likewise replace the subscripts "1j" with "1" and go to the next step.

[Step2] Do the same procedure as Step 1 for the output y_2(k) and y_{M2}(k), the following equations can be obtained.

$$\mathbf{z^{f2j}y_2(k) = Ca_{2j}x(k) + Da_{2j}u(k)}$$
$$\mathbf{z^{f2j}y_{M2}(k) = Ca_{M2j}x_M(k) + Da_{M2j}u_M(k)} \quad (16)$$

[Step3] When if $\mathbf{Da_{2j}}$=/= α $\mathbf{Da_1}$, replace the subscripts "2j" with "2" and do the same procedure as Step 2 for y_3(k) and y_{M3}(k). Where α is constant.

When if $\mathbf{Da_{2j}}$= α $\mathbf{Da_1}$, set the following outputs and do the same procedure from Step 2.

$$\mathbf{-\alpha z^{f1}y_1(k) + z^{f2}y_2(k)}$$
$$\mathbf{-\alpha z^{f1}y_{M1}(k) + z^{f2}y_{M2}(k)} \quad (17)$$

[Step4] By repeating the above procedure to the last outputs y_p(k) and y_{Mp}(k), the following equations can be obtained.

$$\mathbf{Na(z)y(k) = Cax(k) + Dau(k)}$$
$$\mathbf{Na(k)y_M(k) = Ca_Mx_M(k) + Da_Mu_M(k)} \quad (18)$$

Where Na(z) is a p x p lower triangular matrix [9-10] in which the diagonal entries are $\mathbf{z^{fi}}$ **(i = 1, 2,..., p)**, and Ca, Da, Ca_M and Da_M are respectively as follows:

$$\mathbf{Ca = [Ca_1, Ca_2,..., Ca_p]^T ; Ca_i = C_i A_D^{fi},}$$
$$\mathbf{Da = [Da_1, Da_2,..., Da_p]^T ; Da_i = C_i A_D^{fi-1}B_D,}$$
$$\mathbf{Ca_M = [Ca_{M1}, Ca_{M2},..., Ca_{Mp}]^T ; Ca_{Mi} = C_{Mi} A_M^{fi}}\ \text{and}$$
$$\mathbf{Da_M = [Da_{M1}, Da_{M2},..., Da_{Mp}]^T ; Da_{Mi} = C_{Mi} A_M^{fi-1}B_M,}$$
(i = 1, 2,..., p)

Then it is clear that Na(z) is a lower triangular matrix because of the procedure in Step 3 which derives the relations between $y_j(k)$ and u(k) with the time shift form of y_j (k).

Using the above relation the following theorem can be obtained.

[Theorem]

If the following condition is satisfied

$$\mathbf{rank(Da) = p}$$

System Σ can be model-matched to Reference System Σ_M by the control law u(k) as

$$\mathbf{u(k) = Da^{-1} \{ -Cax(k) + Na(z)y_M(k)\}} \tag{19}$$

(Proof)

Define the output error e(k) as

$$\mathbf{e(k) = y_M(k) - y(k)} \tag{20}$$

The following relation can be obtained using Eq.(18)-(19)

$$\mathbf{Na(z)e(k) = 0} \tag{21}$$

And for the condition x(0)=0 and $x_M(0)$=0, the following model matching can be achieved.

$$\mathbf{y(k) = y_M(k) \quad for\ k \geq 0} \tag{22}$$

6. SIMULATION[4]

6.1 Determination of the control law

Perform the procedure in Section 5 for the discrete time form of Eq.(9). At first the following equation can be obtained from the relations, $\mathbf{y_1(k) = \dot{v}(k)/g}$ and Eq.(8).

$$\mathbf{y_1(k) = \{a_{11}v(k) + a_{12}(k)\omega(k) + b_{11}(k)\delta_f(k) + b_{12}(k)\delta_r(k)\}/g} \tag{23}$$

Next left-multiply $\mathbf{y_2(k) = V\omega(k)/g}$ by time shift operator z, the following equation between y_2(k) and the inputs can be obtained.

$$\mathbf{zy_2(k) = V\{ a_{d21}v(k) + a_{d22}(k)\omega(k) + b_{d21}(k)\delta_f(k) + b_{d22}(k)\delta_r(k) \} /g} \tag{24}$$

Where the signals having subscription d are the entries in Eq.(8) calculated based on Eq.(11) in sampling time ΔT.
From Eq.(23) and Eq.(24), the following equation can be obtained.

$$\begin{bmatrix} \delta_f(k) \\ \delta_r(k) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{d21} & b_{d22} \end{bmatrix}^{-1} \left\{ -\begin{bmatrix} a_{11} & a_{12} \\ a_{d21} & a_{d22} \end{bmatrix}\begin{bmatrix} v(k) \\ \omega(k) \end{bmatrix} + \begin{bmatrix} g & 0 \\ 0 & gz/V \end{bmatrix}\begin{bmatrix} y_{M1}(k) \\ y_{M2}(k) \end{bmatrix} \right\} \tag{25}$$

6.2 Condition for simulation

Consider the following discrete time 2nd-order system as the reference model.

$$\mathbf{y_M(k) = diag\{0.0676/(z^2+1.74z+0.08076)\}u_M(k)} \tag{26}$$

Where $\mathbf{y_M(k) \in R^2}$; $\mathbf{y_{M1}(k) + y_{M2}(k) = D_M^*(k) \in R}$, the damping ratio and the natural frequency are $\mathbf{\zeta = 0.9}$ and $\mathbf{\omega n = 5.2[rad/s]}$. And the reference model inputs are given respectively as follows:

$$\mathbf{[u_{M1}(t), u_{M2}(t)] = [\ 0.05, 0.05] \quad (0 \leq t < 5)}$$
$$\mathbf{[u_{M1}(t), u_{M2}(t)] = [-0.05, -0.05] \quad (5 \leq t)} \tag{27}$$

Moreover the running condition [7] is given as follows:

[Running condition]

M = 1050[kg], I = 1330[kg m²],
l_f = 1.37[m], l_r = 1.46[m],
C_f = 25400[N/rad], C_r = 37800[N/rad] and
V = 60[km/h]

The simulation results are shown in Fig.4-7.

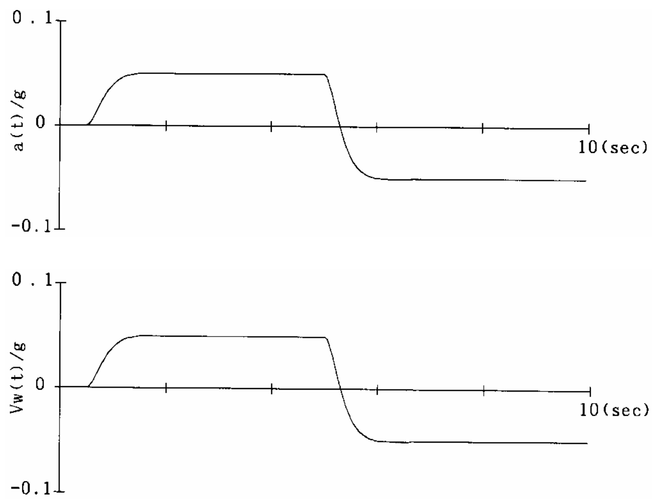

Fig.4 Responses of the outputs: $\dot{v}$(k)/g & Vω(k)/g

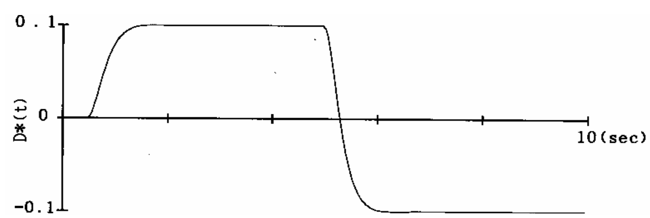

Fig.5 D*(t) response

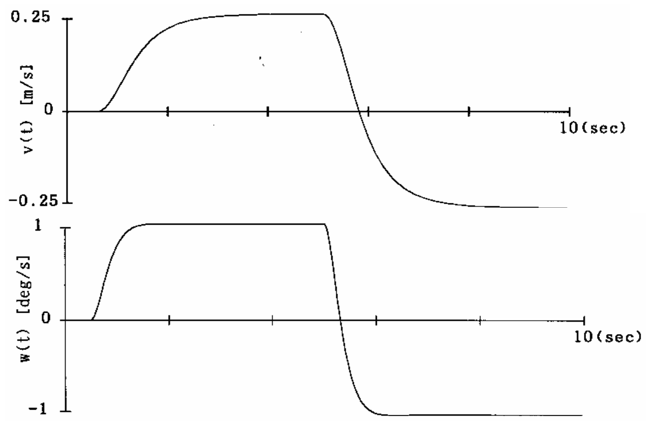

Fig.6 Responses of the state variables: v(t) & ω (t)

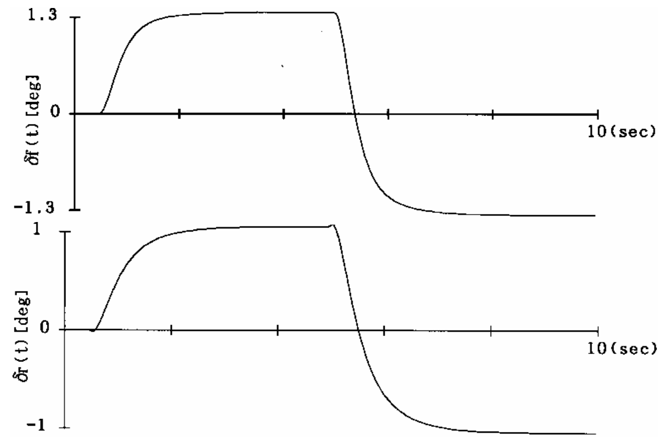

Fig.7 The control inputs: $\delta_f(t)$ & $\delta_r(t)$

[Comment]

Fig.4 shows that the outputs $\dot{v}$ (t)/g and V ω (t)/g perfectly matched to the reference model outputs $y_{M1}(t)$ and $y_{M2}(t)$ on the running condition at **V = 60km/h,** the feasibility of our proposed control method can be verified.

The sum of each output in Fig.4 is D*(t) response in Fig.5. Therefore as to the transient response of this steering system which affects the handling quality and ride comfortableness, if necessary, the driver should change the transfer function in Eq.(26) so as to obtain the good response which satisfies the driver.

Also it is possible to give the good response which satisfies the driver by dividing the reference value D_M*(t) given by the driver in other ratio and providing them for each reference model input. Moreover as the special motion mode, only the yaw rate control (running on the arbitrary turning radius) or only the lateral-directional acceleration control (crab-wise) can be realized.

7. CONCLUSION

In this study adopting the other point of view for D*(t), an MIMO model matching control system for 4WS is constructed using $\dot{v}$ (t)/g and V ω (t)/g as the outputs and the forward/rear wheel angles as the control inputs. Each output is controlled to match each reference model provided by driver, as the result D* response which is the sum of each output converges within the range of some envelope. And the design method used in Section 5 makes the determination of control law easy and can be available for nonlinear control systems.

At the end of paper the feasibility of proposed control system was verified by numerical simulations.

As the subjects that should be studied, the simulations for the crab-wise motion etc., the synthesis of this control system in the cases that the all state variables cannot be measured and the plant parameters varies [11-12], are remained.

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