A Note on Positive Invariant Set for Linear Uncertain Discrete-Time Systems

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Abstract: This paper gives some sufficient conditions for a given polyhedral set which is represented as a set of linear inequalities to be positive $\mathcal{D}$-invariant for uncertain linear discrete-time systems in the case such that the systems matrices depend linearly on uncertain parameters whose ranges are given intervals. Further, the results will be applied to uncertain linear continuous systems in the sense of the above by using Euler approximation.

Keywords: Positive $\mathcal{D}$-invariance, discrete-time systems, uncertain systems

1. Introduction

A positive invariant set is a fundamental concept and is very important to investigate stability analysis and the design of constrained controllers for linear dynamical systems. Further, positive $\mathcal{D}$-invariant set is also important one which has the property that the state trajectories of the systems still remain in a given set for all disturbances. In 1988 some algebraic conditions on positive invariance of various types of polyhedral sets for linear discrete-time systems were studied by Bitsoris [1]. After that Blanchini[2] investigated necessary and sufficient conditions for a given polyhedral region to be positive $\mathcal{D}$-invariant for both discrete-time and continuous-time systems. Recently, from the practical viewpoint, Lin, Zhai and Antsaklis [4] investigated positive $\mathcal{D}$-invariant set for linear systems whose system's matrix contains uncertain parameters, and its application to set valued observer for a class of uncertain linear systems ware also studied. However, the obtained result does not contain the conditions of structual properties of uncertain systems.

In this paper, uncertain linear discrete-time systems in the case in which the systems matrices depend linearly on uncertain parameters whose ranges are given intervals are considered and some sufficient conditions for the polyhedral set which is represented as a set of linear inequalities to be positive $\mathcal{D}$-invariant are given for the uncertain linear discrete-time systems. Further, the results will be applied to uncertain linear continuous systems in the sense of the above by using Euler approximation. Finally, an illustrative example will be given.

2. Positive $\mathcal{D}$-invariant Set

Consider the following discrete-time systems of the form

$$x(t+1) = Ax(t) + Ed(t) \tag{1}$$

where $x(t) \in \mathcal{X} := R^n$ is the state, $d(t) \in \mathcal{D} \subset R^m$ is the disturbance and A and E are constant real matrices of appropriate dimensions. And $\mathcal{D}$ is a compact and convex set containing the origin.

Definition 1: [2] A set $\mathcal{S}$ ($\subset \mathcal{X}$) is said to be a positive $\mathcal{D}$-invariant (PDI) for system (1) if for every initial state $x(0) \in \mathcal{S}$ and every disturbance sequence $d(t) \in \mathcal{D}$ ($t = 0, 1, \ldots$), then the state trajectories $x(t) \in \mathcal{S}$ for $t > 0$. In the particuler case, $\mathcal{D} = \{0\}$, the positive $\mathcal{D}$-invariant set is said to be a positive invariant (PI). ■

Now, we have the definition of the polyhedral set and is used throughtout this study .
Definition 2: A set $\mathcal{S}$ is polyhedral if it is the intersection of a finite number of closed half-spaces, and is represented by

$$\mathcal{S} = \{x \in \mathcal{X} \mid f_\mu^\top x \le \theta_\mu, \quad \mu = 1, \ldots, s\}$$

where $f_\mu \in R^n$ and $\theta_\mu \in R$ is scalar, or equivalently

$$\mathcal{S} = \{x \in \mathcal{X} \mid Fx \le \theta\}$$

where $F \in R^{s\times n}$ is a constant matrix, $\theta \in R^s$ is a vector and $\le$ is with respect to componentwise. Further, the set of vertices of $\mathcal{S}$ is denoted by $vert\{S\}$. ■

Throughtout this investigation, it is assumed that the polyhedral set $\mathcal{S}$ and disturbance region $\mathcal{D}$ are convex and compact polyhedrons containing the origin in their interiors, respectively.

Then, the following theorem holds.
Theorem 1: [2] The polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invariant for system (1) if and only if for every $v_k \in vert\{\mathcal{S}\}$ ($k = 1, \ldots, r$) and $w_h \in vert\{\mathcal{D}\}$ ($h = 1, \ldots, l$),

$$Av_k + Ew_h \in \mathcal{S}$$

which implies

$$f_\mu^\top (Av_k + Ew_h) \le \theta_\mu \quad (\mu = 1, \ldots, s).$$ ■

3. Main Results

This section gives the main results on positive $\mathcal{D}$-invariance for uncertain linear discrete-time systems.

Consider the following uncertain discrete-time systems :

$$x(t+1) = (A_0 + \sum_{i=1}^{p} a_i A_i)x(t) + (E_0 + \sum_{j=1}^{q} e_j E_j)d(t) \quad (2)$$

where $x(t) \in \mathcal{X} \subset R^n$ is the state, $d(t) \in \mathcal{D} \subset R^m$ is the disturbance and A_i $(i = 0,\ldots,p)$ and E_j $(j = 0,\ldots,q)$ are constant real matrices of appropriate dimentions. And a_i $(i = 1,\ldots,p)$ and e_j $(j = 1,\ldots,q)$ are scalar uncertain parameters which are in given intervals as follows.

$$\underline{a}_i \le a_i \le \overline{a}_i \quad (i = 1,\ldots,p), \quad \underline{e}_j \le e_j \le \overline{e}_j \quad (j = 1,\ldots,q)$$

where, $\underline{a}_i$, $\overline{a}_i$, $(i = 1,\ldots,p)$ and $\underline{e}_j$, $\overline{e}_j$,$(j = 1,\ldots,q)$ are known parameters. Then, the uncertain parameters are represented as the following convex combinations :

$$a_i = \alpha_i \underline{a}_i + (1-\alpha_i)\overline{a}_i \ \ (i = 1,\ldots,p), \quad (3)$$
$$e_j = \epsilon_j \underline{e}_j + (1-\epsilon_j)\overline{e}_j \quad (j = 1,\ldots,q) \quad (4)$$

for $0 \le \alpha_i \le 1$ $(i = 1,\ldots,p)$ and $0 \le \epsilon_j \le 1$ $(j = 1,\ldots,q)$. Now, it follows from (3) and (4) that the system (2) can be written by

$$x(t+1) = (A_0 + \sum_{i=1}^{p} \alpha_i \underline{a}_i A_i + \sum_{i=1}^{p}(1-\alpha_i)\overline{a}_i A_i)x(t)$$
$$+(E_0 + \sum_{j=1}^{q} \epsilon_j \underline{e}_j E_j + \sum_{j=1}^{q}(1-\epsilon_j)\overline{e}_j E_j)d(t). \quad (5)$$

Noticing that

$$1 = \frac{\sum_{i=1}^{p}\alpha_i + \sum_{i=1}^{p}(1-\alpha_i) + \sum_{j=1}^{q}\epsilon_j + \sum_{j=1}^{q}(1-\epsilon_j)}{p+q},$$

the system (5) can be represented as

$$x(t+1) = \sum_{i=1}^{p}\alpha_i\{(\frac{1}{p+q}A_0 + \underline{a}_i A_i)x(t) + \frac{1}{p+q}E_0 d(t)\}$$
$$+ \sum_{i=1}^{p}(1-\alpha_i)\{(\frac{1}{p+q}A_0 + \overline{a}_i A_i)x(t) + \frac{1}{p+q}E_0\}d(t)$$
$$+ \sum_{j=1}^{q}\epsilon_j\{\frac{1}{p+q}A_0 x(t) + (\frac{1}{p+q}E_0 + \underline{e}_j E_j)d(t)\}$$
$$+ \sum_{j=1}^{q}(1-\epsilon_j)\{\frac{1}{p+q}A_0 x(t) + (\frac{1}{p+q}E_0 + \overline{e}_j E_j)d(t)\}. \quad (6)$$

Then, the following main result can be obtained.

Theorem 2: If there exist parameters $\lambda_{\underline{a}_i}$, $\lambda_{\overline{a}_i}$ $(i = 1,\ldots,p)$ and $\lambda_{\underline{e}_j}$, $\lambda_{\overline{e}_j}$ $(j = 1,\ldots,q)$ such that for every $v_k = vert\{\mathcal{S}\}$ $(k = 1,\ldots,r)$ and $w_h = vert\{\mathcal{D}\}$ $(h = 1,\ldots,l)$ the following inequalities :

$$f_\mu^\top\{(\frac{1}{p+q}A_0 + \underline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\} \le \lambda_{\underline{a}_i}\theta_\mu,$$
$$f_\mu^\top\{(\frac{1}{p+q}A_0 + \overline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\} \le \lambda_{\overline{a}_i}\theta_\mu,$$
$$f_\mu^\top\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \underline{e}_j E_j)w_h\} \le \lambda_{\underline{e}_j}\theta_\mu,$$
$$f_\mu^\top\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \overline{e}_j E_j)w_h\} \le \lambda_{\overline{e}_j}\theta_\mu$$
$$(\mu = 1,\ldots,s)$$

and

$$\sum_{i=1}^{p}\lambda_{\underline{a}_i} + \sum_{i=1}^{p}\lambda_{\overline{a}_i} + \sum_{j=1}^{q}\lambda_{\underline{e}_j} + \sum_{j=1}^{q}\lambda_{\overline{e}_j} \le 1$$

are satisfied, then the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invariant for the uncertain system (2).

Proof. Suppose that the stated above conditions are satisfied. Then, the following inequalities are satisfied.

$$f_\mu^\top\ [\sum_{i=1}^{p}\alpha_i\{(\frac{1}{p+q}A_0 + \underline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\}\ ]$$
$$+f_\mu^\top\ [\sum_{i=1}^{p}(1-\alpha_i)\{(\frac{1}{p+q}A_0 + \overline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\}\ ]$$
$$+f_\mu^\top\ [\sum_{j=1}^{q}\epsilon_j\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \underline{e}_j E_j)w_h\}\ ]$$
$$+f_\mu^\top\ [\sum_{j=1}^{q}(1-\epsilon_j)\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \overline{e}_j E_j)w_h\}\ ]$$
$$\le \sum_{i=1}^{p}\alpha_i\lambda_{\underline{a}_i}\theta_\mu + \sum_{i=1}^{p}(1-\alpha_i)\lambda_{\overline{a}_i}\theta_\mu$$
$$+ \sum_{j=1}^{q}\epsilon_j\lambda_{\underline{e}_j}\theta_\mu + \sum_{j=1}^{q}(1-\epsilon_j)\lambda_{\overline{e}_j}\theta_\mu$$
$$= \{\sum_{i=1}^{p}\alpha_i\lambda_{\underline{a}_i} + \sum_{i=1}^{p}(1-\alpha_i)\lambda_{\overline{a}_i}$$
$$+ \sum_{j=1}^{q}\epsilon_j\lambda_{\underline{e}_j} + \sum_{j=1}^{q}(1-\epsilon_j)\lambda_{\overline{e}_j}\}\theta_\mu$$
$$\le \theta_\mu \quad (\mu = 1,\ldots,s). \quad (7)$$

Then, it follows from (2), (6) and (7) that

$$f_\mu^\top\ [\ (A_0 + \sum_{i=1}^{p}a_i A_i)v_k + (E_0 + \sum_{j=1}^{q}e_j E_j)w_h\ ]$$
$$= f_\mu^\top\ [\sum_{i=1}^{p}\alpha_i\{(\frac{1}{p+q}A_0 + \underline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\}$$
$$+ \sum_{i=1}^{p}(1-\alpha_i)\{(\frac{1}{p+q}A_0 + \overline{a}_i A_i)v_k + \frac{1}{p+q}E_0 w_h\}$$
$$+ \sum_{j=1}^{q}\epsilon_j\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \underline{e}_j E_j)w_h\}$$
$$+ \sum_{j=1}^{q}(1-\epsilon_j)\{\frac{1}{p+q}A_0 v_k + (\frac{1}{p+q}E_0 + \overline{e}_j E_j)w_h\}\ ]$$
$$\le \theta_\mu \quad (\mu = 1,\ldots,s)$$

for all uncertain parameters a_i and e_j. Hence, it follows from Theorem 1 that the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invairiant for the uncertain system (2). ■

4. Consideration for Continuous Systems

Consider the following uncertain linear continuous systems :

$$\dot{x}(t) = (A_0 + \sum_{i=1}^{p} a_i A_i)x(t) + (E_0 + \sum_{j=1}^{q} e_j E_j)d(t) \tag{8}$$

where $x(t) \in R^n$ is the state, $d(t) \in \mathcal{D} \subset R^m$ is the disturbance, and $A_i \quad (i = 0, \ldots, p)$ and $E_j \quad (j = 0, \ldots, q)$ are constant real matrices of appropriate dimentions. And $a_i \ (i = 1, \ldots, p)$ and $e_j \ (j =, \ldots, q)$ are scalar uncertain parameters satisfying the following conditions.

$$\underline{a}_i \le a_i \le \overline{a}_i \quad (i = 1, \ldots, p), \quad \underline{e}_j \le e_j \le \overline{e}_j \quad (j = 1, \ldots, q)$$

where $\underline{a}_i$, $\overline{a}_i$ $(i = 1, \ldots, p)$ and $\underline{e}_j$, $\overline{e}_j$ $(j = 1, \ldots, q)$ are known parameters.

Now, the following theorem was given by Blanchini.
Theorem 3: [2] Assume that $a_i = 0 \ (i = 1, \ldots, p)$ and $e_j = 0 \ (j = 1, \ldots, q)$ for the systems (8). Then, the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invariant for the system (8) if and only if there exists a $\tau > 0$ such that for all $v_k \in vert\{\mathcal{S}\} \quad (k = 1, \ldots, r)$ and $w_h \in vert\{\mathcal{D}\} \quad (h = 1, \ldots, l)$,

$$v_k + \tau(A_0 v_k + E_0 w_h) \in \mathcal{S}$$

which implies

$$f_\mu^\top (v_k + \tau(A_0 v_k + E_0 w_h)) \le \theta_\mu \quad (\mu = 1, \ldots, s). \qquad \blacksquare$$

Now, since the uncertain parameters are represented as the following convex combinations :

$$a_i = \alpha_i \underline{a}_i + (1 - \alpha_i)\overline{a}_i, \quad i = 1, \ldots p \tag{9}$$
$$e_j = \epsilon_j \underline{e}_j + (1 - \epsilon_j)\overline{e}_j, \quad j = 1, \ldots q \tag{10}$$

where $0 \le \alpha_i \le 1 \quad (i = 1, \ldots, p)$ and $0 \le \epsilon_j \le 1 \quad (j = 1, \ldots, q)$, it follows from (9) and (10) that the system (8) can be written by

$$\begin{aligned}\dot{x}(t) = (A_0 + \sum_{i=1}^{p} \alpha_i \underline{a}_i A_i + \sum_{i=1}^{p} (1 - \alpha_i)\overline{a}_i A_i)x(t) \\ + (E_0 + \sum_{j=1}^{q} \epsilon_j \underline{e}_j E_j + \sum_{j=1}^{q} (1 - \epsilon_j)\overline{e}_j E_j)d(t). \end{aligned} \tag{11}$$

Then, the Euler approximation with τ for the system (11) is represented as

$$\begin{aligned} x(t+1) = \{I + \tau(A_0 + \sum_{i=1}^{p} \alpha_i \underline{a}_i A_i + \sum_{i=1}^{p} (1 - \alpha_i)\overline{a}_i A_i)\}x(t) \\ + \tau(E_0 + \sum_{j=1}^{q} \epsilon_j \underline{e}_j E_j + \sum_{j=1}^{q} (1 - \epsilon_j)\overline{e}_j E_j)d(t). \end{aligned} \tag{12}$$

Noticing that

$$1 = \frac{\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{p} (1 - \alpha_i) + \sum_{j=1}^{q} \epsilon_j + \sum_{j=1}^{q} (1 - \epsilon_j)}{p + q},$$

the system (12) can be written by

$$\begin{aligned} x(t+1) = \\ & \sum_{i=1}^{p} \alpha_i \left[\{ \frac{1}{p+q}(I + \tau A_0) + \tau \underline{a}_i A_i \} x(t) + \frac{1}{p+q}\tau E_0 d(t) \right] \\ & + \sum_{i=1}^{p} (1 - \alpha_i) \left[\{ \frac{1}{p+q}(I + \tau A_0) + \tau \overline{a}_i A_i \} x(t) \right. \\ & \left. \qquad + \frac{1}{p+q}\tau E_0 d(t) \right] \\ & + \sum_{j=1}^{q} \epsilon_j \left[\frac{1}{p+q}(I + \tau A_0)x(t) + \{ \frac{1}{p+q}\tau E_0 + \underline{e}_j E_j \} d(t) \right] \\ & + \sum_{j=1}^{q} (1 - \epsilon_j) \left[\frac{1}{p+q}(I + \tau A_0)x(t) \right. \\ & \left. \qquad + \{ \frac{1}{p+q}\tau E_0 + \overline{e}_j E_j \} d(t) \right]. \end{aligned} \tag{13}$$

Then, we have the following second main result.
Theorem 4: If there exeist parareters $\tau > 0$, $\lambda_{\underline{a}_i}$, $\lambda_{\overline{a}_i}$ $(i = 1, \ldots, p)$ and $\lambda_{\underline{e}_j}$, $\lambda_{\overline{e}_j}$ $(j = 1, \ldots, q)$ such that for every $v_k = vert\{\mathcal{S}\} \quad (k = 1, \ldots, r)$ and $w_h = vert\{\mathcal{D}\} \quad (h = 1, \ldots, l)$ such that the following inequalities :

$$f_\mu^\top \left[\{ \frac{1}{p+q}(I + \tau A_0) + \tau \underline{a}_i A_i \} v_k + \frac{1}{p+q}\tau E_0 w_h \right] \le \lambda_{\underline{a}_i} \theta_\mu,$$

$$f_\mu^\top \left[\{ \frac{1}{p+q}(I + \tau A_0) + \tau \overline{a}_i A_i \} v_k + \frac{1}{p+q}\tau E_0 w_h \right] \le \lambda_{\overline{a}_i} \theta_\mu,$$

$$f_\mu^\top \left[\frac{1}{p+q}(I + \tau A_0) v_k + \{ \frac{1}{p+q}\tau E_0 + \underline{e}_j E_j \} w_h \right] \le \lambda_{\underline{e}_j} \theta_\mu,$$

$$f_\mu^\top \left[\frac{1}{p+q}(I + \tau A_0) v_k + \{ \frac{1}{p+q}\tau E_0 + \overline{e}_j E_j \} w_h \right] \le \lambda_{\overline{e}_j} \theta_\mu$$

$$(\mu = 1, \ldots, s)$$

and

$$\sum_{i=1}^{p} \lambda_{\underline{a}_i} + \sum_{i=1}^{p} \lambda_{\overline{a}_i} + \sum_{j=1}^{q} \lambda_{\underline{e}_j} + \sum_{j=1}^{q} \lambda_{\overline{e}_j} \le 1,$$

are satisfied, then the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invariant for the uncertain system (8).

Sketch of Proof. Suppose that the stated above conditions are satisfied. Then, the following equations can be easily obtained.

$$\begin{aligned} & f_\mu^\top [\{ I + \tau(A_0 + \sum_{i=1}^{p} a_i A_i)\} v_j + \tau(E_0 + \sum_{j=1}^{q} e_j E_j) w_h] \\ & = f_\mu^\top \left[\{ I + \tau(A_0 + \sum_{i=1}^{p} \alpha_i \underline{a}_i A_i + \sum_{i=1}^{p} (1 - \alpha_i)\overline{a}_i A_i)\} x(t) \right. \\ & \qquad \left. + \tau(E_0 + \sum_{j=1}^{q} \epsilon_j \underline{e}_j E_j + \sum_{j=1}^{q} (1 - \epsilon_j)\overline{e}_j E_j) d(t) \right] \\ & = f_\mu^\top \left[\sum_{i=1}^{p} \alpha_i [\{ \frac{1}{p+q}(I + \tau A_0) + \tau \underline{a}_i A_i \} v_k + \frac{1}{p+q}\tau E_0 w_h] \right. \\ & \qquad + \sum_{i=1}^{p} (1 - \alpha_i) [\{ \frac{1}{p+q}(I + \tau A_0) + \tau \overline{a}_i A_i \} v_k \end{aligned}$$

$$+\frac{1}{p+q}\tau E_0 w_h]$$

$$+\sum_{j=1}^{q}\epsilon_j[\frac{1}{p+q}(I+\tau A_0)v_k+\{\frac{1}{p+q}\tau E_0+\underline{e}_j E_j\}w_h]$$

$$+\sum_{j=1}^{q}(1-\epsilon_j)[\frac{1}{p+q}(I+\tau A_0)v_j$$

$$+\{\frac{1}{p+q}\tau E_0+\overline{e}_j E_j\}w_h]\ ]$$

$$\leq \theta_\mu \quad (\mu=1,\ldots,s)$$

for all uncertain parameters a_i and e_j. Hence, it follows form Theorem 3 that the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invarinant for the system (8). ■

5. A Numerical Example

Consider the following continuous linear uncertain system :

$$\dot{x}(t)=\left\{\begin{bmatrix}-0.6 & 0\\ 0 & -0.6\end{bmatrix}+a_1\begin{bmatrix}0 & 0\\ 0.1 & 0\end{bmatrix}+a_2\begin{bmatrix}0 & 0\\ 0 & 0.1\end{bmatrix}\right\}x(t)+\left\{\begin{bmatrix}-0.1\\ 0.1\end{bmatrix}+e_1\begin{bmatrix}0.1\\ 0\end{bmatrix}\right\}d(t) \quad (14)$$

where

$$-0.5\leq a_1\leq 0.5,\quad -0.2\leq a_2\leq -1.5,\quad -0.1\leq e_1\leq 0.1$$

and disturbance region is as follows.

$$\mathcal{D}=\{d(t)\in R^1 \mid -0.1\leq d(t)\leq 0.1\}$$

Now, we consider the polyhedral set S :

$$S=\left\{\begin{bmatrix}x_1\\ x_2\end{bmatrix}\in R^2 \mid \begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\leq\begin{bmatrix}1\\ 1\\ 1\\ 1\end{bmatrix}\right\}$$

$$=\left\{\begin{bmatrix}x_1\\ x_2\end{bmatrix}\in R^2 \mid -1\leq x_1\leq 1,\quad -1\leq x_2\leq 1\right\}.$$

Then, the Euler approximated systems for the system (14) is given by

$$x(t+1)=\left[I+\tau\left\{\begin{bmatrix}-0.6 & 0\\ 0 & -0.6\end{bmatrix}+a_1\begin{bmatrix}0 & 0\\ 0.1 & 0\end{bmatrix}+a_2\begin{bmatrix}0 & 0\\ 0 & 0.1\end{bmatrix}\right\}\right]x(t)+\tau\left\{\begin{bmatrix}-0.1\\ 0.1\end{bmatrix}+e_1\begin{bmatrix}0.1\\ 0\end{bmatrix}\right\}d(t).$$

Now, choose parameters $\tau=1$ and

$$\lambda_{\underline{a}_1}=0.1876,\quad \lambda_{\underline{a}_2}=0.1376,\quad \lambda_{\overline{a}_1}=0.1867,$$
$$\lambda_{\overline{a}_2}=0.1367,\quad \lambda_{\underline{e}_1}=0.1377,\quad \lambda_{\overline{e}_1}=0.1367.$$

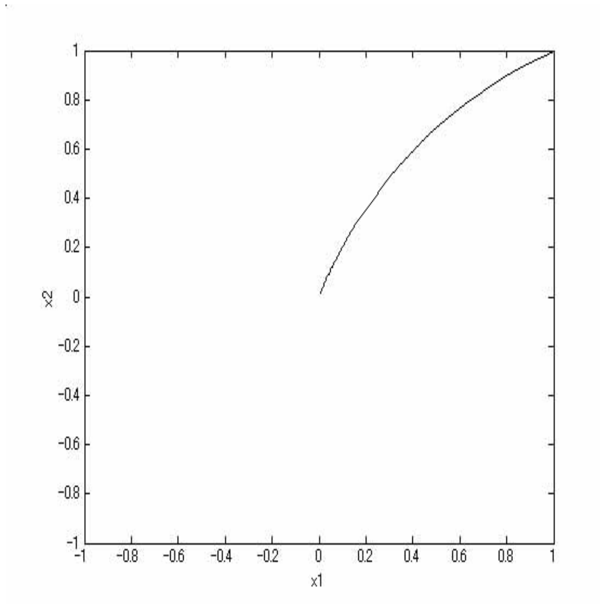

Fig. 1. Trajectory of the system (14)

Then, since the conditions of Theorem 4 are satisfied, we can see that the polyhedral set $\mathcal{S}$ is positive $\mathcal{D}$-invarinat for the uncertain system (14). The figure 1 shows that the state trajectory of the system (14) still remain in the polyhedral set $\mathcal{S}$ for uncertain parameters $a_1=0.5$, $a_2=-1.5$, $e_1=0.1$ and an initial state $x(0)=[1,1]^\top\in\mathcal{S}$.

6. Concluding Remarks

In this paper, some sufficient conditions for a given polyhedral set which is represented as a set of linear inequalities to be positive $\mathcal{D}$-invariant for uncertain linear discrete-time systems were given. Further, the obtained results were also applied to uncertain linear continuous-time systems by using Euler approximation. Finally, we need to investigate how to choose parameters λ satisfying the conditions in the main theorems as a future study.

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