Mixed H_2/H_∞ Finite Memory Controls for Output Feedback Controls of Discrete-time State-Space Systems

Choon Ki Ahn, Soo Hee Han, and Wook Hyun Kwon

School of Electrical Engineering & Computer Science, Seoul National University, Seoul , 151-742, Korea
(Tel: +82-2-880-7314; http://cisl.snu.ac.kr; Email:hironaka@cisl.snu.ac.kr)

Abstract: In this paper, a new type of output feedback control, called a H_2/H_∞ finite memory control (FMC), is proposed for deterministic state space systems. Constraints such as linearity, unbiasedness property, and finite memory structure with respect to an input and an output are required in advance to design H_2/H_∞ FMC in addition to the performance criteria in both H_2 and H_∞ sense. It is shown that H_2, H_∞, and mixed H_2/H_∞ FMC design problems can be converted into convex programming problems written in terms of linear matrix inequalities (LMIs) with some linear equality constraints. Through simulation study, it is illustrated that the proposed H_2/H_∞ FMC is more robust against uncertainties and faster in convergence than the existing H_2/H_∞ output feedback control schemes.

Keywords: Mixed H_2/H_∞ finite memory control, Unbiasedness property, Receding horizon, Output feedback control

1. Introduction

The mixed H_2/H_∞ output feedback control utilizes measurements to generate the control that satisfies both H_2 and H_∞ specifications given in terms of bounds [1], [2], [3], [4]. These controls can be synthesized by combining a control part and an estimation part or by generating the control from dynamic models. In case of the latter, the transfer function from the measurement to the control has an infinite impulse response (IIR).

In signal processing area, the system with finite impulse response (FIR) is preferable since the accumulation of undesirable effects can be avoided due to a finite memory structure. Thus, there have been a wide of researches on characteristics and efficient implementations for FIR systems. As FIR systems, FIR filters have been widely used and investigated, which were also proposed in state space models as a substitute of Kalman filter [5], [6]. In case of controls, there are some trials to apply the finite memory structure as the FIR system to the design of the control according to the linear quadratic Gaussian performance criterion for continuous-time systems [7] and discrete-time systems [8], respectively. However, there are no results for H_∞ performance criterion and mixed H_2/H_∞ performance criterion. In this paper, H_∞ and mixed H_2/H_∞ output feedback controls with finite memory structure will be proposed.

H_∞ and mixed H_2/H_∞ output feedback controls with finite memory structure can be represented using measurements and inputs during a finite time, i.e., a horizon, as

$$u_k = \sum_{i=k-N_f}^{k-1} H_{k-i} y_i + \sum_{i=k-N_f}^{k-1} L_{k-i} u_i \qquad (1)$$

for some gains H_i and L_i. Note that even though the control (1) uses the finite measurements and inputs on the recent time interval as FIR filters, this is not of the FIR form. So this kind of the control will be called finite memory controls (FMC) rather than FIR controls. In this paper, H_i and L_i will be determined to minimize the H_∞ performance criterion under the upper bounded H_2 performance and the FMC with these H_i and L_i will be called the mixed H_2/H_∞ FMC. The proposed H_2/H_∞ FMC is both unbiased and optimal *by design* for the given performance criterion. The '*by design*' means that the unbiased property and optimality are built into the proposed FMC during its design simultaneously. In addition, the centering concept [9] of the control to the optimal state state feedback control makes BMI problem change into LMI problem so that it gets easier to solve the mixed H_2/H_∞ FMC problem.

This paper is organized as follows. In Section 2, some definitions and problem statement are given. In Section 3, H_2, H_∞, and mixed H_2/H_∞ FMC problems are solved in terms of linear matrix inequalities (LMIs). In Section 4, numerical example is given. Finally, conclusion is stated in Section 5.

2. Problem Formulation

Consider a linear discrete-time state space model:

$$x_{k+1} = Ax_k + Bu_k + Gw_k, \qquad (2)$$

$$y_k = Cx_k + Dw_k \qquad (3)$$

$$z_k = D_1 x_k + D_2 u_k \qquad (4)$$

where $x_k \in \Re^n$, $u_k \in \Re^l$, $y_k \in \Re^q$, and $z_k \in \Re^p$ are the state, the input, the measurement, and the controlled signal, respectively. Note that $D_1^T D_2 = 0$, $D_2^T D_2 = I$, $DG^T = 0$, and $DD^T = I$.

The system (2)-(3) will be represented in a batch form on the time interval $[k-N_f, k]$ called the filter horizon. On the horizon $[k-N_f, k]$, measurements are expressed in terms of the state x_k at the time k and inputs as follows:

$$Y_{k-1} = \bar{C}_{N_f} x_k + \bar{B}_{N_f} U_{k-1} + \bar{G}_{N_f} W_{k-1} + \bar{D}_{N_f} W_{k-1} \qquad (5)$$

where

$$Y_{k-1} \triangleq [y_{k-N_f}^T \;\; y_{k-N_f+1}^T \;\; \cdots \;\; y_{k-1}^T]^T, \qquad (6)$$

$$U_{k-1} \triangleq [u_{k-N_f}^T \;\; u_{k-N_f+1}^T \;\; \cdots \;\; u_{k-1}^T]^T, \qquad (7)$$

$$W_{k-1} \triangleq [w_{k-N_f}^T \; w_{k-N_f+1}^T \; \cdots \; w_{k-1}^T]^T,$$

and $\bar{C}_{N_f}$, $\bar{B}_{N_f}$, $\bar{G}_{N_f}$ are obtained from

$$\bar{C}_i \triangleq \begin{bmatrix} CA^{-i} \\ CA^{-i+1} \\ CA^{-i+2} \\ \vdots \\ CA^{-1} \end{bmatrix} = \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix} A^{-1}, \quad (8)$$

$$\bar{B}_i \triangleq -\begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-i}B \\ 0 & CA^{-1}B & \cdots & CA^{-i+1}B \\ 0 & 0 & \cdots & CA^{-i+2}B \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}B \end{bmatrix} = \begin{bmatrix} \bar{B}_{i-1} & -\bar{C}_{i-1}A^{-1}B \\ 0 & -CA^{-1}B \end{bmatrix}, \quad (9)$$

$$\bar{G}_i \triangleq -\begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-i}G \\ 0 & CA^{-1}G & \cdots & CA^{-i+1}G \\ 0 & 0 & \cdots & CA^{-i+2}G \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix} = \begin{bmatrix} \bar{G}_{i-1} & -\bar{C}_{i-1}A^{-1}G \\ 0 & -CA^{-1}G \end{bmatrix}, \quad (10)$$

$$\bar{D}_i \triangleq \big[\mathrm{diag}(\overbrace{D \; D \; \cdots \; D}^{i})\big] = \big[\mathrm{diag}(\bar{D}_{i-1}, D)\big], \quad 1 \le i \le N_f.$$

The mixed H_2/H_∞ FMC with FIR structure can be expressed as a linear function of the finite measurements Y_{k-1} and inputs U_{k-1} on the filter horizon $[k-N_f, k]$ as follows:

$$u_k \triangleq HY_{k-1} + LU_{k-1} \quad (11)$$

where H and L are gain matrices. It is desirable that the FMC (11) should be unbiased from the desirable optimal state feedback control as

$$u_k = u_k^*, \qquad \forall w_k = 0 \quad (12)$$

Denote $T_{ew}(z)$ as the transfer function from the exogenous input w_k to the difference $e_k = u_k - u_k^*$ between the input u_k and the optimal state feedback law u_k^*. H and L of the mixed H_2/H_∞ FMC are determined by optimization problem based on the following performance criterions:

$$\min_{H,L} \gamma$$

subject to

$$\sup_{w_k} \frac{\sum_{k=0}^{\infty} ||z_k||_2^2}{\sum_{k=0}^{\infty} ||w_k||_2^2} < \gamma^2, \quad \|T_{ew}(z)\|_2 < \beta \quad (13)$$

In the next section, we will present the solution of H_2 and H_∞ FMC problems.

3. Mixed H_2/H_∞ FMC

3.1. H_2 FMC

For $w_k = 0$, we obtain from (5)

$$u_k = HY_{k-1} + LU_{k-1} = H\bar{C}_N x_k + H\bar{B}_N U_{k-1} + LU_{k-1}.$$

The optimal state feedback control under the following LQ criterion

$$\sum_{j=0}^{N_c-1} \big[x_{k+j}^T Q x_{k+j} + u_{k+j}^T R u_{k+j}\big] + x_{k+N_c}^T F x_{k+N_c} \quad (14)$$

is given by

$$u_k^* = -R^{-1}B^T[I + K_1BR^{-1}B^T]^{-1}K_1Ax_k = -[R + B^TK_1B]^{-1}B^TK_1Ax_k, \quad (15)$$

where K_i is given by

$$K_i = A^TK_{i+1}A - A^TK_{i+1}B[R + B^TK_{i+1}B]^{-1}B^T \times K_{i+1}A + Q \quad (16)$$

$$= A^TK_{i+1}[I + BR^{-1}B^TK_{i+1}]^{-1}A + Q \quad (17)$$

with the boundary condition

$$K_{N_c} = F. \quad (18)$$

Therefore, the following constraints on H and L are required for (12) to hold:

$$H\bar{C}_{N_f} = -[R + B^TK_1B]^{-1}B^TK_1A$$

$$H\bar{B}_{N_f} = -L. \quad (19)$$

From (19), the FMC in (11) is rewritten into

$$u_k = H(Y_{k-1} - \bar{B}_{N_f}U_{k-1}) \quad (20)$$

$$H\bar{C}_{N_f} = -[R + B^TK_1B]^{-1}B^TK_1A \quad (21)$$

The constraint $H\bar{C}_{N_f} = -[R + B^TK_1B]^{-1}B^TK_1A$ will be called the quasi-deadbeat constraint in the sense that it is a deadbeat constraint for the nominal system without the exogenous input $w_k = 0$, but may not be a deadbeat constraint for the system (2) and (3) with nonzero exogenous input. Next, we derive the transfer function $T_{ew}(z)$. Exogenous input w_k satisfies the following state model on W_{k-1}

$$W_k = A_uW_{k-1} + B_uw_k, \quad (22)$$

where

$$A_u = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{pN_f \times pN_f} \quad (23)$$

$$B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I \end{bmatrix} \in R^{pN_f \times p} \quad (24)$$

It follows from (5) that

$$Y_{k-1} - \bar{B}_{N_f} U_{k-1} = \bar{C}_{N_f} x_k + (\bar{G}_{N_f} + \bar{D}_{N_f}) W_{k-1}. \quad (25)$$

Pre-multiplying (25) by H and using the constraint $H\bar{C}_{N_f} = -[R + B^T K_1 B]^{-1} B^T K_1 A$ gives

$$e_k = u_k - u_k^* = H(\bar{G}_{N_f} + \bar{D}_{N_f}) W_{k-1}. \quad (26)$$

From (41) and (40), we can obtain $T_{ew}(z)$ as follows:

$$T_{ew}(z) = H(\bar{G}_{N_f} + \bar{D}_{N_f})(zI - A_u)^{-1} B_u. \quad (27)$$

Based on $T_{ew}(z)$, we have the following theorem for H_2 FMC:

Theorem 1: Assume that the following LMI problem is feasible:

$$\min_{F,W} \ \text{tr}(W) \text{ subject to}$$

$$\begin{bmatrix} W & S \\ S^T & I \end{bmatrix} > 0.$$

where

$$S = FM(\bar{G}_{N_f} + \bar{D}_{N_f}) + H_0(\bar{G}_{N_f} + \bar{D}_{N_f}) \quad (28)$$

, $H_0 = -[R + B^T K_1 B]^{-1} B^T K_1 A (\bar{C}_{N_f}^T \bar{C}_{N_f})^{-1} \bar{C}_{N_f}^T$, and M^T is the bases of the null space of $\bar{C}_{N_f}^T$. Then the optimal gain matrix of the H_2 FMC of the form (20) is given by

$$H = FM + H_0.$$

Proof. The constraint $H\bar{C}_{N_f} = -[R + B^T K_1 B]^{-1} B^T K_1 A$ is required for the H_2 FMC to be of the form (20). H_2 norm of the transfer function $T_{ew}(z)$ in (27) is obtained by

$$\|T_{ew}(z)\|_2^2 = \text{tr}\big(H(\bar{G}_{N_f} + \bar{D}_{N_f}) M (\bar{G}_{N_f} + \bar{D}_{N_f})^T H^T\big),$$

where

$$M = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i.$$

Since $A_u^i = 0$ for $i \geq N_f$, we obtain

$$M = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i = \sum_{i=0}^{N_f - 1} A_u^i B_u B_u^T (A_u^T)^i = I.$$

Thus we have

$$\|T_{ew}(z)\|_2^2 = \text{tr}\big(H(\bar{G}_{N_f} + \bar{D}_{N_f})(\bar{G}_{N_f} + \bar{D}_{N_f})^T H^T\big). \quad (29)$$

Introduce a matrix variable W such that

$$W > H(\bar{G}_{N_f} + \bar{D}_{N_f})(\bar{G}_{N_f} + \bar{D}_{N_f})^T H^T. \quad (30)$$

Then $\text{tr}(W) > \|T_{ew}(z)\|_2^2$. By Schur complement, (30) is equivalently changed into

$$\begin{bmatrix} W & H(\bar{G}_{N_f} + \bar{D}_{N_f}) \\ (\bar{G}_{N_f} + \bar{D}_{N_f})^T H^T & I \end{bmatrix} > 0. \quad (31)$$

Hence, by minimizing $\text{tr}(W)$ subject to $H\bar{C}_{N_f} = -[R + B^T K_1 B]^{-1} B^T K_1 A$ and the above LMI, we can obtain the optimal gain matrix H for the H_2 FMC. The equality constraint $H\bar{C}_{N_f} = -[R + B^T K_1 B]^{-1} B^T K_1 A$ can be eliminated by computing the null space of $\bar{C}_N^T$. All solutions to the equality constraint $H\bar{C}_{N_f} = -[R + B^T K_1 B]^{-1} B^T K_1 A$ are parameterized by

$$H = FM + H_0, \quad (32)$$

where F is a matrix containing the independent variables. Replacing H by $FM + H_0$, the LMI condition in (31) is changed into the one in the Theorem 1. This completes the proof. ■

3.2. H_∞ FMC

For the system transfer function

$$G(z) \triangleq \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(zI - A)^{-1} B + D,$$

we introduce the well-known bounded real lemma.

Lemma 1: (Bounded real lemma) Let $\gamma > 0$. The following two conditions are equivalent:

(1) $\|G(z)\|_\infty < \gamma$.

(2) There exists an $X > 0$ such that

$$\begin{bmatrix} -X & XA & XB & 0 \\ A^T X & -X & 0 & C^T \\ B^T X & 0 & -\gamma I & D^T \\ 0 & C & D & -\gamma I \end{bmatrix} < 0.$$

If we use the bounded real lemma to derive H_∞ FMC, the resultant matrix inequality can be described by BMI(Binear Matrix Ineqality) with respect to H and L. Some complicated BMI should be solved numerically. In the following, instead of this BMI form, a LMI form of H_∞ FMC will be proposed by introducing a centering technique.

The state-feedback solution to the infinite horizon H_∞ performance criterion of (13) is given in a form of

$$u_k^* = -B^T P[I + (BB^T - \gamma^2 GG^T)P]^{-1} A x_k \quad (33)$$

$$w_k^* = \gamma^{-2} G^T P[I - \gamma^{-2} GG^T P]^{-1} \times (Ax_k + Bw_k) \quad (34)$$

where P is the solution to the following algebraic H_∞ Riccati equation:

$$P = A^T P[I + (BB^T - \gamma^{-2} GG^T)P]^{-1} A + D_1^T D_1.$$

Using w_k^* and the state space (2)-(3), the following new state space is obtained :

$$x_{i+1} = Ax_i + Bu_i + Gw_i$$

$$
\begin{aligned}
&= Ax_i + Bu_i + G(w_i - w_i^*) + Gw_i^* \\
&= \left[I + \gamma^{-2}GG^TP[I - \gamma^{-2}GG^TP]^{-1}\right] \\
& \quad (Ax_i + Bu_i) + G(w_i - w_i^*) \\
&= [I - \gamma^{-2}GG^TP]^{-1}(Ax_i + Bu_i) \\
&+ \; G\Delta w_i \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
y_i &= Cx_i + Dw_i \\
&= Cx_i + Dw_i \\
&- \; \gamma^{-2}DG^TP[I - \gamma^{-2}GG^TP]^{-1}(Ax_i + Bu_i) \\
&= Cx_i + D\Delta w_i \qquad (36)
\end{aligned}
$$

where $\Delta w_i = w_i - w_i^*$. We can treat Δw_i as disturbance. The control problem based on (13) is reduced to the estimation problem as (42). In other words, all that remain to do is to estimate u_k^* in (33).

The new state space (35)-(36) can be represented in a batch form on the time interval $[k - N_f, k]$

$$
Y_{k-1} = \bar{C}_{N_f}^* x_k + \bar{B}_{N_f}^* U_{k-1} + (\bar{G}_{N_f}^* + \bar{D}_{N_f})\Delta W_{k-1} \qquad (37)
$$

where

$$
\Delta W_{k-1} \triangleq \begin{bmatrix} w_{k-N_f} - w_{k-N_f}^* \\ w_{k-N_f+1} - w_{k-N_f+1}^* \\ w_{k-N_f+2} - w_{k-N_f+2}^* \\ \vdots \\ w_{k-1} - w_{k-1}^* \end{bmatrix}
$$

$\bar{C}_{N_f}^*$, $\bar{B}_{N_f}^*$, and $\bar{G}_{N_f}^*$ are defined by replacing A and B with $[I - \gamma^{-2}GG^TP]^{-1}A$ and $[I - \gamma^{-2}GG^TP]^{-1}B$. Without disturbance, u_k in (11) is represented as

$$
\begin{aligned}
u_k &= HY_{k-1} + LU_{k-1} \\
&= H\bar{C}_{N_f}^* x_k + H\bar{B}_{N_f}^* U_{k-1} + LU_{k-1}.
\end{aligned}
$$

u_k can be centered to the optimal state feedback control by setting

$$
H\bar{C}_{N_f}^* = -B^TP[I + (BB^T - \gamma^2 GG^T)P]^{-1}A \qquad (38)
$$

$$
H\bar{B}_{N_f}^* = -L. \qquad (39)
$$

Using the constraints (38) and (39) gives

$$
e_k \triangleq u_k - u_k^* = H(\bar{G}_{N_f}^* + \bar{D}_{N_f}^*)\Delta W_{k-1}. \qquad (40)
$$

Disturbance Δw_k satisfies the following state model on ΔW_{k-1}:

$$
\Delta W_k = A_u \Delta W_{k-1} + B_u \Delta w_k \qquad (41)
$$

where A_u and B_u are given in (23) and (24). In the new state space, the performance criterion (13) can be changed to

$$
\sup_{w_k} \frac{||u_k - u_k^*||_2}{||w_k - w_k^*||_2} < \gamma^2. \qquad (42)
$$

From (41) and (40), we can obtain a transfer function $T_{e\Delta w}(z)$ from the disturbance Δw_k to the estimation error e_k as follows:

$$
T_{e\Delta w}(z) = H(\bar{G}_{N_f}^* + \bar{D}_{N_f}^*)(zI - A_u)^{-1}B_u. \qquad (43)
$$

Using Lemma 1, we can obtain the LMI for FMC satisfying the H_∞ performance .

Theorem 2: Assume that the following LMI is satisfied for $X > 0$ and F:

$$
\min_{X>0,F} \gamma_\infty
$$

subject to

$$
\begin{bmatrix} -X & XA_u & XB_u & 0 \\ A_u^TX & -X & 0 & \Xi^T \\ B_u^TX & 0 & -\gamma_\infty I & 0 \\ 0 & \Xi & 0 & -\gamma_\infty I \end{bmatrix} < 0
$$

where

$$
\begin{aligned}
\Xi &= FM^*(\bar{G}_{N_f}^* + \bar{D}_{N_f}^*) + H_0^*(\bar{G}_{N_f}^* + \bar{D}_{N_f}^*) \qquad (44) \\
H_0^* &= -B^TP[I + (BB^T - \gamma^2 GG^T)P]^{-1}A \\
&\times \; (\bar{C}_{N_f}^{*T}\bar{C}_{N_f}^*)^{-1}\bar{C}_{N_f}^{*T} \qquad (45)
\end{aligned}
$$

, and M^{*T} is the bases of the null space of $\bar{C}_{N_f}^{*T}$. Then, the gain matrices of the H_∞ FMC of the form (20) are given by

$$
H = FM^* + H_0^*, \quad L = -H\bar{B}_{N_f}^*.
$$

Proof. According to Lemma1, the condition $\|T_{e\Delta w}(z)\|_\infty < \gamma_\infty$ is equivalent to

$$
\begin{bmatrix} -X & XA_u & XB_u & 0 \\ {A_u}^TX & -X & 0 & (\bar{G}_{N_f}^* + \bar{D}_{N_f}^*)^TH^T \\ {B_u}^TX & 0 & -\gamma_\infty I & 0 \\ 0 & H(\bar{G}_{N_f}^* + \bar{D}_{N_f}^*) & 0 & -\gamma_\infty I \end{bmatrix} < 0
$$

The equality constraint $H\bar{C}_{N_f}^* = -B^TP[I + (BB^T - \gamma^2 GG^T)P]^{-1}A$ can be eliminated in the exactly same way as in H_2 FMC. ■

3.3. Mixed $H_2/H_\infty FMC$

Let's define γ_2^* to be the $\|T_{ew}(z)\|_2^2$ due to the optimal H_2 FMC. From the previous two subsections, it is so clear how to formulate the H_2/H_∞ FMC problem. Thus, we have the following theorem for the mixed H_2/H_∞ FMC:

Theorem 3: Assume that the following LMI problem is feasible:

$$
\min_{W,X>0,F} \gamma_\infty \quad \text{subject to}
$$

$$
\text{tr}(W) < \alpha\gamma_2^*, \quad \text{where} \quad \alpha > 1
$$

$$
\begin{bmatrix} W & S \\ S^T & I \end{bmatrix} > 0,
$$

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ A_u^T X & -X & 0 & \Xi^T \\ B_u^T X & 0 & -\gamma_\infty I & 0 \\ 0 & \Xi & 0 & -\gamma_\infty I \end{bmatrix} < 0$$

where S, Ξ, and H_0^* are defined in (28), (44), and (45), respectively. M^{*T} is the bases of the null space of $\bar{C}_{N_f}^{*T}$. Then, the gain matrix of the H_2/H_∞ FMC of the form (20) is given by

$$H = FM^* + H_0^*.$$

Proof. So clear, hence omitted. The above mixed H_2/H_∞

FMC problem allows us to design the optimal FMC with respect to the H_∞ norm while assuring a prescribed performance level in the H_2 sense. By adjusting $\alpha > 0$, we can trade off the H_∞ performance against the H_2 performance.

Remark 1: Optimal H_2 FMC can be obtained analytically from [8], [10]

$$H_B = -\mathcal{K}_\infty(\bar{\mathcal{C}}^T_{\mathcal{N}_\{} \div^{-\infty}_{\mathcal{N}_\{} \bar{\mathcal{C}}_{\mathcal{N}_\{})^{-\infty} \bar{\mathcal{C}}^T_{\mathcal{N}_\{} \div^{-\infty}_{\mathcal{N}_\{}.$$

Thus we have

$$\gamma_2^* = \mathrm{tr}(H_B \Xi_{N_f} H_B^T),$$

where $\mathcal{K}_\infty$ and Ξ_{N_f} are obtained from

$$\begin{aligned} \mathcal{K}_\infty &= R^{-1}B^T[I + K_1 B R^{-1} B^T]^{-1} K_1 A \\ &= [R + B^T K_1 B]^{-1} B^T K_1 A, \\ \Xi_i &\triangleq (\bar{G}_i + \bar{D}_i)(\bar{G}_i + \bar{D}_i)^T \\ &= \bar{G}_i \bar{G}_i^T + \bar{D}_i \bar{D}_i^T \\ &= \begin{bmatrix} \bar{G}_{i-1}\bar{G}_{i-1}^T + \bar{D}_{i-1}\bar{D}_{i-1}^T & 0 \\ 0 & I \end{bmatrix} \\ &+ \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix} A^{-1} G G^T A^{-T} \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix}^T \\ &= \begin{bmatrix} \Xi_{i-1} & 0 \\ 0 & I \end{bmatrix} \\ &+ \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix} A^{-1} G G^T A^{-T} \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix}^T \qquad (46) \end{aligned}$$

for $1 \le i \le N_f$.

4. Numerical Example

To illustrate the validity of the proposed FMC, numerical example to compare the proposed H_2/H_∞ FMC and the existing H_2/H_∞ output feedback control of [3], [4] is given for the following linear discrete-time invariant state-space model which has actual temporary uncertainty:

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0.33 + 2\delta_k & 0.01 + \delta_k \\ 0.01 & 0.9 + 3\delta_k \end{bmatrix} x_k + \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix} u_k \\ &+ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} w_k \\ y_k &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \end{bmatrix} w_k \\ z_k &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_k \end{aligned}$$

where δ_k is a model uncertain parameter which is assumed to satisfy

$$\delta_k = \begin{cases} 0.1, & 100 \le k \le 150 \\ 0, & \text{otherwise} \end{cases}.$$

Figures 1 and 2 compare the state trajectories of x_1 and x_2, respectively, in case that the exogenous input w_k is given by

$$w_k = \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix}, \text{ where } w_{1k} \sim (0,1),\ w_{2k} \sim (0,1).$$

From this simulation result, it is clearly shown that the proposed mixed H_2/H_∞ FMC is more robust against to the uncertainty and faster in convergence. Therefore, it is expected that the proposed H_2/H_∞ FMC can be usefully used in real applications.

5. Conclusion

In this paper, a new type of control called the mixed H_2/H_∞ FMC was proposed for discrete-time state space signal models. The control problem has been formulated in terms of linear matrix inequalities (LMIs). The proposed control scheme enables us to consider both the H_2 and the H_∞ performances. The proposed mixed H_2/H_∞ FMC is linear with the most recent finite measurements and inputs, and has the unbiasedness property from the optimal state feedback control. Furthermore, due to the FIR structure of FMC, the proposed scheme is believed to be robust against temporary modelling uncertainties or numerical errors, while other output feedback control method with an IIR structure such as dynamic output feedback control or observer based control may show poor robustness in these cases. The proposed H_2/H_∞ FMC will be useful for many pratical control problems where signals are represented by state space models.

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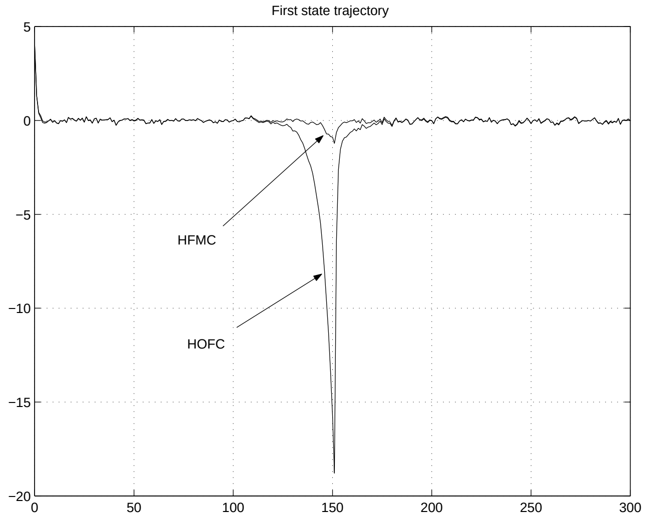

Fig. 1. Trajectory of x_1

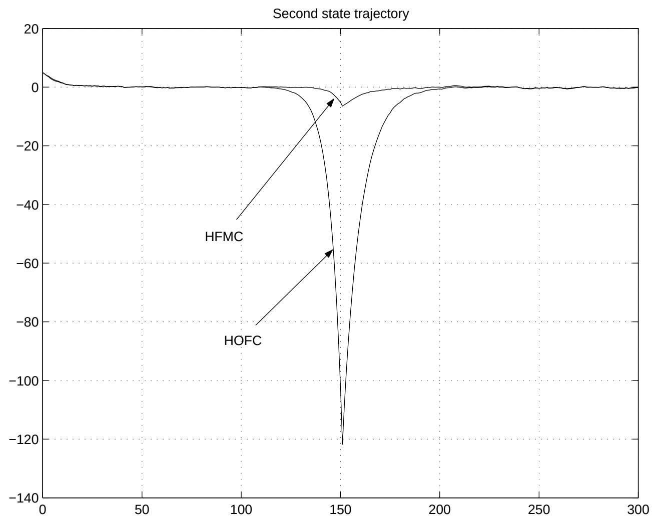

Fig. 2. Trajectory of x_2

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