Discrete-Time Feedback Error Learning with PD Controller

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Abstract: In this study, the basic motor control system had been investigated. The Discrete-Time Feedback Error Learning (DTFEL) method is used to control this system. This method is anologous to the original continuous-time version Feedback Error Learning(FEL) control which is proposed as a control model of cerebellum in the field of computational neuroscience. The DTFEL controller consists of two main parts, a feedforward controller part and a feedback controller part. Each part will deals with different control problems. The feedback controller deals with robustness and stability, while the feedforward controller deals with response speed. The feedforward controller, used to solve the tracking control problem, is adaptable. To make such the tracking perfect, the adaptive law is designed so that the feedforward controller becomes an inverse system of the controlled plant. The novelty of FEL method lies in its use of feedback error as a teaching signal for learning the inverse model. The PD control theory is selected to be applied in the feedback part to guarantee the stability and solve the robust stabilization problems. The simulation of each individual part and the integrated one are taken to clarify the study.

Keywords: Discrete-time system, Feedback Error Learning, PD controller, strictly positive real, learning control.

1. Introduction

The production machines in the industrial have been played a significant role. They have been improved in order to increase both quantity and quality of the production. They are high-cost, over functional and, due to the used of static PID controller [3], have moderate performance. Meanwhile, there are some locally made ones but they are low-performance even though they have moderate cost. Nowadays, digital controllers are replacing the analog one. This is due to the rapid development in Microelectronics making digital electronics components much cheaper with very high performance adequate to apply to the control applications. Digital control theory and many control algorithms, which are digital based, have also been developed and adopt to the real systems.

The objective of this study is to find the cheap controller but can achieve the desired system performance.

For simplicity, the simple DC-servo motor, instead of the expensive CNC machine, is used as a controlling plant. Normally, one controller scheme alone can not response so well. So more than one controller schemes are usually proposed.

Feedback Error Learning (FEL) [2, 4, 5, 6, 8, 9, 11, 12] itself consists of more than one control mechanisms.

The FEL novel architecture combines learning and control efficiently. The novelty of FEL method lies in its use of feedback error as a teaching signal for learning the inverse model, which is essentially new in control literature. Originally, FEL is adopted from the concept of brain motor control as stated in [6].

In this study, the mathematical knowledge which is useful for analyzing the DTFEL system, is briefly discussed in the first part. Then, the stability of the DTFEL system is analyzed. After that, the PD controller is discussed. Next, the simulation results from the study will be demonstrated. Finally, the conclusion of this study will be shown.

2. Mathematical Preliminaries

In this section, the mathematical requirement to analyze the DTFEL in the next section is discussed. The main and most important area is to study the strictly positive real system. Also, there are many new theorems proved in this section.

Consider the linear discrete-time varying system given by

$$\begin{aligned} x(k+1) &= A(k)x(k)+B(k)u(k), \\ y(k) &= C(k)x(k)+D(k)u(k), \end{aligned} \tag{1}$$

with $A(k)$, $B(k)$, $C(k)$, and $D(k)$ are appropriately dimensioned matrices. The pulse-transfer matrix of this system is $H(z) = C(zI-A)B+D$.

The definition of positive real and strictly positive real is given as following definitions.

Definition 1: [10] A square matrix $H(z)$ of real rational functions is a positive real (p.r.) matrix if

(d1) $H(z)$ has elements analytic in $|z|>1$.

(d2) $H^T(z^*)+H(z)$ is positive, semidefinite and Hermitian for $|z|>1$.

In case that $H(z)$ has some simple poles on $|z|=1$, condition (d2) can be replaced by

(d3) The poles of the elements of $H(z)$ on $|z|=1$ are simple and the associated residue matrixes of $H(z)$ at these poles are 0,

(d4) $H(e^{j\theta})+H^T(e^{-j\theta})$ is a positive semidefinite Hermitian matrix for all real θ for which $H(e^{j\theta})$ exists.

Definition 2: [10] A rational transfer matrix $H(z)$ is a strictly positive real (s.p.r.) matrix if $H(\rho z)$ is p.r. for some $0<\rho<1$.

Given definition 2, a necessary and sufficient condition in the frequency domain for s.p.r. transfer matrices in the class $\mathfrak{H}$ can be defined as following.

Definition 3: [10] An $n \times n$ rational matrix $H(z)$ is said to belong to class $\mathfrak{H}$ if $H(z) + H^T(z^{-1})$ has rank m almost everywhere in the complex z-plane.

Theorem 1: [10] Consider the $m \times m$ rational matrix $H(z) \in \mathfrak{H}$ given in Eq. 2. Then $H(z)$ is a s.p.r. matrix if and only if

(a) All elements of $H(z)$ are analytic in $|z| \geqslant 1$,

(b) $H(e^{j\theta}) + H^T(e^{-j\theta}) > 0$, $\forall \theta \in [0, 2\pi]$.

A system with a s.p.r. transfer matrix has many advantage characteristics which are very important in adaptive control. The following lemma mathematically represents the characteristics necessary for proving the system stability.

Lemma 1 (Discrete-time version of Kalman-Yakubovich-Popov) [10] Assume that the rational transfer matrix $H(z)$ has poles that lie in $|z| < \gamma$, where $0 < \gamma < 1$ and (A, B, C, D) is a minimal realization of $H(z)$. Then $H(\gamma z)$ is s.p.r., if and only if real matrices $P = P^T > 0$, Q and K exist such that

$$\begin{aligned} A^T PA - P &= -QQ^T - (1-\gamma^2)P, \\ APB &= C^T - QK, \\ K^T K &= D + D^T - B^T PB. \end{aligned}$$

Remark

If $L(z)$ is a stable transfer function, there exists sufficiently large K such that $\frac{\alpha}{K}(L(z)+K)^{-1}$ is s.p.r.

The following lemmas present some useful control theorems adopted from Lemma 1.

Lemma 2: [1] Define $\psi(k_1, k_0)$ as the state-transition matrix corresponding to $A(k)$ in Eq. (1), i.e., $\psi(k_1, k_0) = \prod_{k=k_0}^{k_1-1} A(k)$. Then if $\|\psi(k_1, k_0)\| \leqslant 1$, $\forall k_1, k_0 \geqslant 0$, the system represented by Eq. (1) is exponentially stable.

Lemma 3: [1] If $A(k) = I - \alpha\phi(k)\phi^T(k)$ in Eq. (1), where $0 < \alpha < 2$ and $\phi(k)$ is a regressor vector of past inputs and outputs, then $\|\phi(k_1, k_0)\| < 1$ is guaranteed if there is an $L > 0$ such that $\sum_{k=k_0}^{k_1+L-1} \phi(k)\phi^T(k) > 0$ for all k. Then Lemma 2 guarantees the exponential stability of the system represented by Eq. (1).

Definition 4: [1] An input sequence $x(k)$ is said to be persistently exciting (PE) if $\gamma > 0$ and an integer $k_1 \geqslant 1$ such that

$$\gamma_{\min}\left[\sum_{k=k_0}^{k_1+L-1} \phi(k)\phi^T(k)\right] > \gamma,\ \forall k_0 \geqslant 0, \qquad (2)$$

where $\gamma_{\min}[P]$ represents the smallest eigenvalue of P.

Note: PE is exactly the stability condition needed in Lemma 3. By using the all previous definitions, lemma, and theorems, the following theorem is established:

Theorem 2: A difference equation

$$z(k+1) = \left(I - \xi(k)L(z)\xi^T(k)\right)z(k) \qquad (3)$$

is asymptotically stable for any time-varying vector $\xi(k)$ which satisfies the PE condition, if $L(z)$ is s.p.r.

Note that a special case of Theorem 2 where $L(z) = 1$ corresponds to Lemma 3.

3. Analysis of the Discrete-Time Feedback Error Learning

3.1. Feedforward adaptive control method Without Feedback Element

The discussion of the feedback error learning method (henceforth, it is simply referred as Kawato scheme), from the viewpoint of adaptive control, is the main objective of this section. Fig. 1 illustrates the block diagram of Kawato scheme. From the previous chapter, the feedforward controller K_2 is chosen to be identical to the inverse P^{-1} of P if P is known. Since P is unknown, some adaptive schemes for K_2 are employed so that K_2 converges to P^{-1}.

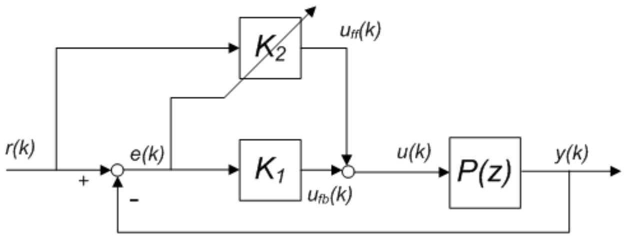

Fig. 1. Discrete-time feedback error learning scheme.

Throughout this chapter, the following assumptions are applied:

Assumptions

(A1) The plant P is stable and has stable inverse P^{-1}.

(A2) The upper bound of the order of P is known.

(A3) $k_0 = \lim_{z\to\infty} P(z)$ is assumed to be positive.

(A4) Input signal is bounded and satisfies the PE condition.

The assumption (A1) is rather restrictive in the context of control system design. This may be relaxed without significant difficulty, but in this study, this assumption is kept in order to focus on the intrinsic nature of the Kawato scheme. In the context of motor control, this assumption is not restrictive because the plant is always a neuro-muscular system with low order. This let the computed torque method, which is essentially equivalent to constructing an inverse model, be applicable.

If k_0 is negative in (A3), the subsequent results are valid by taking $-P(z)$ instead of $P(z)$. Hence, (A3) is relaxed to the assumption that the sign of the high frequency gain is known. For the sake of the simplicity of exposition, however, (A3) is retained. From the assumption (A4), it is obvious that $\xi(k)$ also satisfies PE condition.

3.2. Parameterization of unknown systems

To handle adaptation, it is important to decide how to parameterize the adaptive system. Throughout this study, the following parameterization of the unknown system Q is utilized:

$$\begin{aligned} \xi_1(k+1) &= F\xi_1(k)+gr(k), & (4)\\ \xi_2(k+1) &= F\xi_2(k)+gu_d(k), & (5)\\ u(k) &= c^T(k)\xi_1(k)+d^T(k)\xi_2(k)+l(k)r(k), & (6)\end{aligned}$$

where F is any stable matrix and g is any vector with $\{F,g\}$ being controllable. The block diagram of this parameterization is shown in Fig. 2.

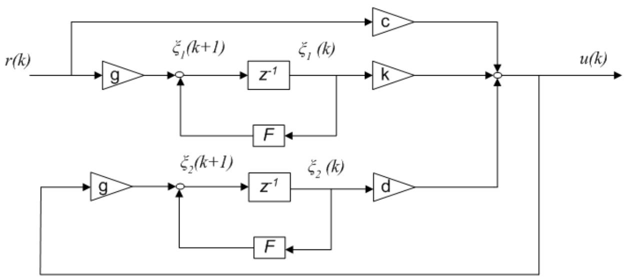

Fig. 2. Parameterization of $K_2(z)$.

In Eqs. (4)∼(6), $c(k)$, $d(k)$, and $l(k)$ are unknown parameters to be estimated, and $u_d(k)$ is the desired output of this system. It is easy to see that taking $u(k)=u_d(k)$ and appropriate selection of parameters $c(k)=c_0$, $d(k)=d_0$, and $l(k)=l_0$ can yield an arbitrary transfer function from $r(k)$ to $u_d(k)$.

The advantage of the parameterization in Eqs. (4)∼(6) is that the unknown parameters enter linearly in the system description. The continuous version of this parameterization was first used in adaptive observer [7].

3.3. Adaptation law

The parameterization of the adaptive feedforward controller K_2 is taken as same as in Eqs. (4)∼(6) . However, instead of $u_d(k)$ in (5), the plant input $u(k)$ is used:

$$\begin{aligned} \xi_1(k+1) &= F\xi_1(k)+gr(k), & (7)\\ \xi_2(k+1) &= F\xi_2(k)+gu_d(k), & (8)\\ u_{ff}(k) &= c^T(k)\xi_1(k)+d^T(k)\xi_2(k)+l(k)r(k), & (9)\\ u(k) &= u_{ff}(k)+K_1e(k), & (10)\end{aligned}$$

where F is stable and $\{F,g\}$ is controllable.

In the ideal situation, K_2 is identical to P^{-1}. In that case, $e(k)=0$, $u(k)=u_{ff}(k)=u_0(k)=P^{-1}(z)r(k)$. The true values c_0, d_0 and k_0 of $c(k)$, $d(k)$ and $k(k)$, respectively, satisfy

$$\frac{l_0+c_0^T\,(zI-F)^{-1}g}{1-d_0^T\,(zI-F)^{-1}g}=P^{-1}(z). \qquad (11)$$

The error signal $e(k)$ is defined as

$$e(k)=r(k)-y(k).$$

The cost function for adaptation is defined as

$$J(k)=\frac{1}{2}\sum_{i=0}^{k}e^2(i). \qquad (12)$$

The unknown parameters $c(k)$, $d(k)$, $l(k)$ must be updated so that the error signal $e(k)$ decreases.

The usual gradient method gives rise to the updating rule. By defining

$$\begin{aligned} \xi(k) &:= [\xi_1(k)^T \quad \xi_2(k)^T \quad r(k)]^T, & (13)\\ \theta(k) &:= [c(k)^T \quad d(k)^T \quad l(k)^T]^T\ , & (14)\end{aligned}$$

the adaptation law of parameters is obtained as

$$\theta(k+1) = \theta(k)+\frac{\alpha}{K_1}e(k)\xi(k). \qquad (15)$$

Note: This is adapted from the continuous-time adaptation algorithm by using gradient method presented by Miyamura [5]. In this case, the desired output $u_d(k)$ must be identical to $u_d(k)=P^{-1}(z)r(k)$ because the feedforward controller is assumed to be an inverse of the plant P. Since P is unknown, so is $u_d(k)$. In Kawato scheme, $u(k)$ is used instead of $u_d(k)$. An essential feature of the present paper lies in the justification of this attempt.

3.4. Convergence proof

The objective of this section is to prove the convergence of the system according to the systems parameterization and adaptation law described previously.

The error signal, defined in previous section, can also be written as

$$e(k)=r(k)-P(z)u(k).$$

Hence,

$$\begin{aligned} u(k) &= u_d(k)-P^{-1}(z)e(k),\\ u_d(k) &= P^{-1}(z)r(k).\end{aligned}$$

Then the adaptive controller is written as

$$\begin{aligned} \xi_1(k+1) &= F\xi_1(k)+gr(k), & (16)\\ \xi_2(k+1) &= F\xi_2(k)+g\left(u_d(k)-P^{-1}(z)e(k)\right), & (17)\\ u_{ff}(k) &= c^T(k)\xi_1(k)+d^T(k)\xi_2(k)+l(k)r(k), & (18)\end{aligned}$$

where

$$\left.\begin{aligned} c(k+1) &= c(k)+\tfrac{\alpha}{K_1}e(k)\xi_1(k),\\ d(k+1) &= d(k)+\tfrac{\alpha}{K_1}e(k)\xi_2(k),\\ l(k+1) &= l(k)+\tfrac{\alpha}{K_1}e(k)r(k).\end{aligned}\right\} \qquad (19)$$

Assume that the true system can be represented as,

$$\begin{aligned} z_1(k+1) &= Fz_1(k)+gr(k), & (20)\\ z_2(k+1) &= Fz_2(k)+gu_d(k), & (21)\\ u_d(k) &= c_0^T(k)z_1(k)+d_0^T(k)z_2(k)+l_0(k)r(k). & (22)\end{aligned}$$

Then,

$$\begin{aligned} u_{ff}(k)-u_d(k) = {} & (c(k)-c_0)^T\,\xi_1(k) & (23)\\ & +(d(k)-d_0)^T\,\xi_2(k)\\ & +(l(k)-l_0)\,r(k)\\ & -d_0^T\,(zI-F)^{-1}gP^{-1}(z)e(k).\end{aligned}$$

Here, the following asymptotic relations are used

$$\begin{aligned} \xi_1(k) &\rightarrow z_1(k),\\ \xi_2(k) &\rightarrow z_2(k)-d_0^T\,(zI-F)^{-1}gP^{-1}(z)e(k).\end{aligned}$$

The relation in Eq. (23) is written as

$$u_{ff}(k) - u_d(k) = \psi(k)^T \xi(k) - d_0^T (zI - F)^{-1} g P^{-1}(z) e(k),$$

where

$$\psi(k) := \theta(k) - \theta_0 = \begin{bmatrix} c(k) - c_0 \\ d(k) - d_0 \\ l(k) - l_0 \end{bmatrix}. \qquad (24)$$

From the relations

$$u(k) = u_{ff}(k) + K_1 e(k),$$

$$-\left[P^{-1}(z)e(k) + K_1 e(k)\right] = \psi(k)^T \xi(k) - d_0^T (zI - F)^{-1} g P^{-1}(z) e(k), \qquad (25)$$

which results in

$$(G(z) + K_1)\, e(k) = \psi(k)^T \xi(k), \qquad (26)$$

$$G(z) := \left(1 - d_0^T (zI - F)^{-1} g\right) P^{-1}(z). \qquad (27)$$

On the other hand, from Eq. (24),

$$\begin{aligned} \psi(k+1) - \psi(k) &= \theta(k+1) - \theta(k) \\ &= \frac{\alpha}{K_1} \xi(k) e(k). \end{aligned} \qquad (28)$$

It should be noted that the relation in Eq. (11) implies that

$$G(z) = l_0 + c_0^T (zI - F)^{-1} g. \qquad (29)$$

Combining Eq. (26) and Eq. (28),

$$\begin{aligned} \psi(k+1) - \psi(k) &= \frac{\alpha}{K_1} \xi(k) e(k) \\ &= \frac{\alpha}{K_1} \xi(k) \left(G(z) + K_1\right)^{-1} \xi(k)^T \psi(k) \\ \psi(k+1) &= \psi(k) - \frac{\alpha}{K_1} \xi(k) \left(G(z) + K_1\right)^{-1} \xi(k)^T \psi(k). \end{aligned} \qquad (30)$$

which is the same form as Eq. (3),i.e.

$$\psi(k+1) = \left(I - \xi(k) L_0(z) \xi(k)^T\right) \psi(k),$$

where $L_0(z)$ is equal to

$$L_0(z) := (G(z) + K_1)^{-1} \frac{\alpha}{K_1}. \qquad (31)$$

According to Theorem 2, the different equation (30) is asymptotically stable, if $L_0(z)$ given by Eq. (31) is s.p.r. K_1 is chosen such that $G(z) + K_1$ is s.p.r. Such K_1 always exists from Definition 2 of s.p.r. (See Remark following Lemma 1). If $G(z) + K_1$ is s.p.r., so is $L_0(z)$. Thus, the following fundamental result have been established:

Theorem 3: Under the assumptions (A1)~(A4), the feedback error learning method represented by Eqs. (16)~(19) is converging, i.e., the controller $K_2(z)$ converges to $P^{-1}(z)$.

4. Discrete-Time Feedback Error Learning with PD Controller

The powerful ability of DTFEL system is the capability to apply another controller to the feedback path to improve the system performance. In this study, the PD controller is chosen to improve the robustness of the system. Specifically, it acts like a disturbance rejector of the system. There are many literatures dealing with discrete-time PD controller [3].

This controller is used due to the concept adopted from the fact that the derivative action of the conventional PID controller can improve the transient response of the system well.

The architecture of the system is the same but, instead of using the constant K_1, the feedback gain is in the form of

$$K_1(z) = K_p + K_d \frac{z-1}{z}.$$

The second term is a discrete-time derivative term consisting of constant derivative gain K_d.

Note that the overall system convergence is still based on DTFEL purely. In another words, the analysis of each can be done separately because the adaptive controller in feedforward path is obviously outside the loop. The only requirement of the system convergence is that $K_1(z)$ is kept to be sufficiently large for all time.

5. Simulation Results

In this section, the simulation results are illustrated to demonstrate the effectiveness of the theoretical results obtained in this study. Three main simulations have been done in order to illustrate the improvement of the system. The pulse-transfer function of the controlling plant is

$$P(z) = \frac{z + 0.2}{z + 0.3}.$$

Note that this plant has a stable inverse. The simulation is done at the sampling period of 10 milliseconds and the adaption of the parameter is done every 100 milliseconds. In Fig. 3, the DTFEL with constant gain K_1 is use as a controller. The tracking performance between the input signal $r(k)$ and the output signal $y(k)$ is shown. The error $e(k) = y(k) - r(k)$ is shown in Fig.

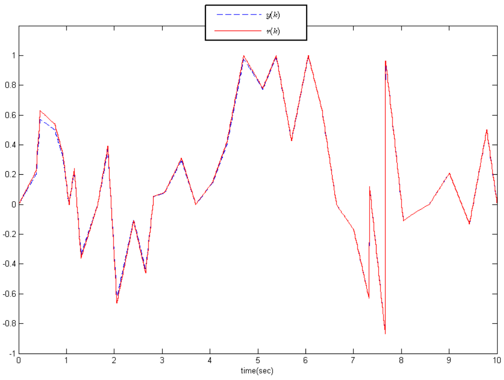

Fig. 3. The simulation result of the DTFEL system with constant feedback gain.

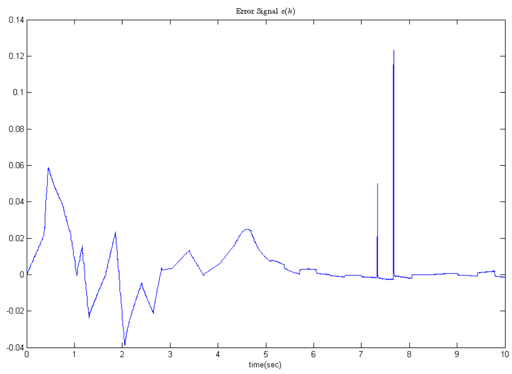

Fig. 4. The error of the DTFEL system with constant feedback gain system.

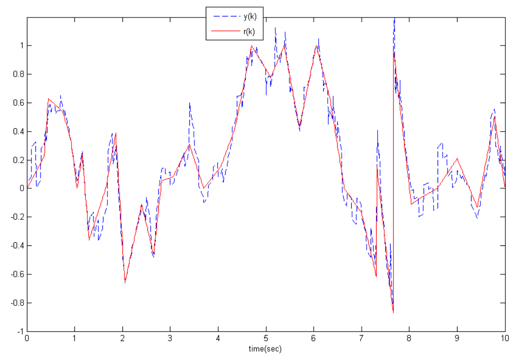

Fig. 5. The simulation result of the DTFEL system with constant feedback gain with system disturbance.

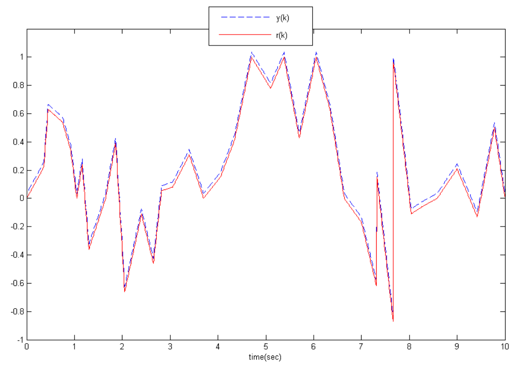

Fig. 6. The simulation result of the DTFEL system with PD controller with system disturbance.

4. These figures show the convergence of the signal and the comparison of the tracking performance of the system before adaptation, from 0 second to about 5.7 second, with after adaptation, from 5.7 second to step 10 second. The learning rate is set to be very low to show the result clearly. In another remaining simulation parts, the higher learning rate is set enabling the system parameters to be converged within 0.5 second.

The second simulation results shown in Fig. 5 showing the response of the same system with additional system disturbance. It is clearly that the system is no longer stable. This can be stabilize by adding the PD controller as the results shown in Fig. 6. It is clearly shown that the low-cost PD controller can improve the tracking performance of the DTFEL very well.

6. Conclusion

In this study, the “Discrete-Time Feedback Error Learning” (DTFEL) is demonstrated. The layout of this paper is following that of the traditional continuous version [5].

The mathematical required to analyze the stability of the DTFEL system, where the controlling has stable inverse, are studied and proved. The simulation results show that if the system have sufficiently large disturbance, the system response is poor. This can be solved by adding another controller in the feedback path. The additional controller, i.e. PD controller is adopted to the system to solve the stability problem of the system. The simulation results guarantee the improvement of the system response. The derivative action of the PD controller acts like a disturbance rejector of the system smoothing the response curve.

The additional PD controller does not affect the convergence of the overall DTFEL system. The analysis of each can be done separately because the adaptive controller in feedforward path is obviously outside the loop.

The auto-tuning PD controller for DTFEL systems, the stability of DTFEL for noninvertible-stable plant and the extension of DTFEL for plant with time-delay will be the future researches. The new algorithms for solving the control problems and improving the system performance are also the open-problems.

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