Two-Step Suboptimal Filters for Linear Dynamic Systems

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Abstract: This paper considers the problem of state estimation in linear continuous-time systems with multi-sensor environment and observation uncertainties. We propose two suboptimal filtering algorithms for these types of systems. The filtering algorithms consist of two steps: The local optimal Kalman estimates are computed at the first step. And, these local estimates are lineally fused at the second step. The implementation of the two-step filtering algorithms needs a lower memory demand than the optimal Kalman and adaptive Lainiotis-Kalman filters. In consequence of parallel structure of the proposed filters, the parallel computers can be used for their design. The examples exhibit the effect of common noise on the performance of fusion of the local Kalman estimates based on observations from different sensors and in the presence of uncertainties.

Keywords: Estimation, Kalman filter, Adaptive filter, Multi-sensor, Data fusion

1. INTRODUCTION

The problem of the system state estimation in linear dynamic systems with uncertain observations is considered. Many techniques are available for the adaptation of systems. Most identification approaches can probably be applied to construct an adaptive mechanism. Among existing methods, we are particularly interested in the partitioned adaptive technique that is mathematically based on Bayesian estimation theory [1], [2].

It is known that the linear filtering problem with unknown parameters, i.e. the adaptive filtering problem, reduces to a nonlinear filtering problem, which has major difficulties in its realization [3]. In particular, it is extremely difficult to assess the effect of approximations made in the suboptimal realization of nonlinear filters. However, partitioned adaptive Lainiotis-Kalman filtering constitutes a partitioning of the original nonlinear filter into a bank or set of much simpler linear Kalman filters. In other words, the optimal mean square estimate of state is given by a weighted sum of the model- or parameter-conditional estimates with weights representing *a posteriori* probabilities of unknown parameter [1], [4], [5]. However the Lainiotis-Kalman filter yields an effective estimation algorithm only for low dimension of the parameter vector, since it requires an evaluation of *a posteriori* probabilities at each time instance.

Here we also consider the estimation problem for dynamic systems with multi-sensory data. In recent years, there has been growing interest to fuse multi-sensory data to increase the accuracy of estimation parameters and system states. This interest is motivated by the availability of different types of sensor which uses various characteristics of the optical, infrared, and electromagnetic spectrums. In many situations, system states or targets are observed by multi-sensors [6], [7], [8]. The overall observations in the estimation process are assigned to a common target as a result of the association process. At that time, we need to know how to combine the local estimates obtained from different types of sensors. The well-known Millman and Bar-Shalom-Campo formulas for fusion of two local estimates are used in the estimation problems [9], [10]. In [11], we have extended these formulas on an arbitrary number of local estimates.

In this paper, we consider two types of dynamic systems. The first is the systems with multi-sensor environment. And, the second is one with observation uncertainties. For these systems, we propose two-step suboptimal filters based on the fusion formula [11], which is applied to fusion of local Kalman filters. The obtained filtering algorithms reduce the computational burden and on-line computational requirements. This has been achieved via the use of a decomposition of the observation vector into a set of subvectors of low dimension. The example demonstrates the efficiency and high-accuracy of the proposed algorithms.

This paper is organized as follows. In Section 2, we consider continuous-time dynamic systems with multi-sensor environment and observation uncertainties. In Section 3, the new suboptimal filters are derived by using the fusion formula. In Section 4, the proposed filters are numerically tested. Finally, Section 5 is the conclusion.

2. LINEAR DYNAMIC SYSTEMS WITH MULTI-SENSOR ENVIRONMENT AND UNCERTAINTIES

Consider a continuous-time linear dynamic system

$$\dot{x}_t = F_t x_t + G_t v_t, \quad t \ge 0. \tag{1}$$

where $x_t \in \mathbf{R}^n$ is the state vector, $\mathbf{R}^n$ is an n-dimensional Euclidean space, $v_t \sim (0, Q_t)$ is Gaussian white noise with zero mean and intensity Q_t. Suppose that the observation system involves N sensors with uncertainties, i.e.,

$$\begin{aligned} y_t^{(1)} &= H_t^{(1)}(\theta) x_t + w_t^{(1)}, \quad y_t^{(1)} \in \mathbf{R}^{m_1}, \\ &\cdots \quad \cdots \\ y_t^{(N)} &= H_t^{(N)}(\theta) x_t + w_t^{(N)}, \quad y_t^{(N)} \in \mathbf{R}^{m_N}, \end{aligned} \tag{2}$$

where $w_t^{(i)} \sim \left(0, R_t^{(i)}(\theta)\right)$. We assume that the initial state $x_0 \sim N(\bar{x}_0, P_0)$, the system noise v_t ,and the observation noises $w_t^{(i)}, \ i = 1, \ldots, N$ are mutually uncorrelated. The system matrixes F_t, G_t, and Q_t are completely known, and the observation matrices $H_t^{(i)}(\theta)$ and $R_t^{(i)}(\theta)$ include an unknown parameter vector $\theta \in \mathbf{R}^{\mathbf{p}}$.

2.1. Optimal Kalman Filter

Let assume that the parameter θ is completely known, i.e., $\theta = \theta^*$, then the Kalman filter (KF) gives optimal mean square estimate $\hat{x}_t^{KF}$ of state x_t based on the *overall* observations Y_t,

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ \vdots \\ y_t^{(N)} \end{bmatrix} = \begin{bmatrix} H_t^{(1)} \\ \vdots \\ H_t^{(N)} \end{bmatrix} x_t + \begin{bmatrix} w_t^{(1)} \\ \vdots \\ w_t^{(N)} \end{bmatrix}. \quad (3)$$

where

$$H_t^{(i)} \equiv H_t^{(i)}(\theta^*), \quad w_t^{(i)} \sim (0, R_t^{(i)}), \quad R_t^{(i)} \equiv R_t^{(i)}(\theta^*). \quad (4)$$

However in the cases of limited computing and communication resources, and large number of sensors, the KF can not produce well-timed results as it simultaneously processes the overall measurements Y_t at each time instant **t** to calculate the current estimate $\hat{x}_t^{KF}$. Therefore, if dimension

$$\dim(Y_t) = m_1 + \cdots + m_N. \quad (5)$$

of the overall observation vector is large, then the KF is unpractical.

2.2. Optimal Adaptive Lainiotis-Kalman Filter

In structure adaptation, two filters are primarily used for the system (1), (2). Both of these filters are based on the Bayesian approach in which the unknown parameter θ is assumed to be random with a *prior* known probability $p(\theta)$. In the first filter, θ is treated as a random constant vector such as $\dot{z}_t = 0, \quad z_0 = \theta$. And the system (1) together with the assumption can be reformulated as the nonlinear model for the composite state vector $[x_t \quad z_t]^T$, and the suboptimal nonlinear filtering procedures can be applied to estimate the composite state $[x_t \quad z_t]^T$,which contains $z_t = \theta$ as its component. However, it is difficult to estimate the effect of approximations made in the suboptimal realization of nonlinear filters. The second filter represents the adaptive Lainiotis-Kalman filter (LKF), which separates the filtering process x_t from the identification of the unknown parameter θ [1], [4], [5]. If it takes only a finite set of values

$$\theta \in \{\theta_1, \ldots, \theta_N\}. \quad (6)$$

then the optimal state estimate $\hat{x}_t^{LKF}$ represents weighting sum of local Kalman filters $\hat{x}_t^{KF}(\theta_i)$ matched to the linear system (1), (2) at fixed $\theta = \theta_i$, i.e.,

$$\hat{x}_t^{LKF} = \sum_{i=1}^{N} \tilde{c}_t^{(i)} \hat{x}_t(\theta_i). \quad (7)$$

where the weights

$$\tilde{c}_t^{(i)} = p(\theta_i | Y_0^t), \quad i = 1, \ldots, N. \quad (8)$$

represent *a posteriori* probabilities of θ_i given $Y_0^t = \{Y_s : 0 \le s \le t\}$. As was already said in the introduction, the LKF is effective only for low dimension of parameter $\theta \in \mathbf{R}^{\mathbf{p}}$, since it requires calculations of *a posteriori* probabilities $p(\theta_i | Y_0^t)$ at each time instance $t > 0$.

In this paper we develop alternative two-stage suboptimal filters (TSSF) for the system (1) with multi-sensor environment (2), when parameter θ is known and unknown. These filters do not require calculations of *a posteriori* probabilities $\tilde{c}_k^{(i)} = p(\theta_i | y^k)$ at each time instance. The obtained filtering algorithms reduce the computational burden and on-line computational requirements. It is achieved via a decomposition of the overall observation vector (3) into a set of subvectors of low dimension.

3. TWO-STEP SUBOPTIMAL FILTERS

The new filtering algorithms consist of two steps. On the first step we determine local Kalman estimates (filters) $\hat{x}_t^{(i)}, i = 1, \ldots, N$. And on the second step, these estimates are fused into overall suboptimal state estimate.

3.1. Local Kalman Filters for Dynamic Systems with Multi-Sensors Environment

Consider dynamic system (1), (3) including N sensors with known parameter $\theta = \theta^*$. According to (1) and (3), we have N dynamic subsystems ($i = 1, \ldots, N$) with state vector $x_t \in \mathbf{R}^{\mathbf{n}}$ and observation subvector $y_t^{(i)} \in \mathbf{R}^{\mathbf{m_i}}$:

$$\dot{x}_t = F_t x_t + G_t v_t, \quad y_t^{(i)} = H_t^{(i)} x_t + w_t^{(i)}. \quad (9)$$

where the number of subsystem i is fixed.

Next, let us denote the estimate of the state x_t based on the individual sensor $y_t^{(i)}$ by $\hat{x}_t^{(i)}$. To find $\hat{x}_t^{(i)}$ we can apply the optimal KF to the subsystem (13) [1], [3]. We have

$$\begin{aligned}\dot{\hat{x}}_t^{(i)} &= F_t\hat{x}_t^{(i)} + P_t^{(ii)}H_t^{(i)^T}R_t^{(i)^{-1}}\left[y_t^{(i)} - H_t^{(i)}\hat{x}_t^{(i)}\right],\\ \dot{P}_t^{(ii)} &= F_tP_t^{(ii)} + P_t^{(ii)}F_t^T - P_t^{(ii)}H_t^{(i)^T}R_t^{(i)^{-1}}H_t^{(i)}P_{.t}^{(ii)} \qquad (10)\\ &+ G_tQ_tG_t^T.\end{aligned}$$

Thus we have N local Kalman estimates (LKEs)

$$\hat{x}_t^{(1)}, \quad \ldots, \quad \hat{x}_t^{(N)}. \qquad (11)$$

based on the individual sensors $y_t^{(1)}$,..., $y_t^{(N)}$, respectively, and corresponding local error covariances (LECs)

$$P_t^{(11)}, \quad \ldots, \quad P_t^{(NN)}. \qquad (12)$$

3.2. Local Kalman Filters for Dynamic Systems with Observation Uncertainties

Consider dynamic system (1), (2) with one sensor ($N=1$) containing unknown parameter θ. We have

$$\dot{x}_t = F_tx_t + G_tv_t, \quad Y_t = H_t(\theta)x_t + w_t. \qquad (13)$$

where $x_t \in \mathbf{R}^n, Y_t = y_t^{(1)} \in \mathbf{R}^{m_1}, H_t(\theta) = H_t^{(1)}(\theta)$,

$\theta \in \{\theta_1, \ldots, \theta_N\}$.

Using the Kalman filter for the system model (13) at the fixed $\theta = \theta_i$, we have N local Kalman estimates

$$\hat{x}_t^{(i)} \equiv \hat{x}_t(\theta_i), \quad i = 1,\ldots,N \qquad (14)$$

and associated local error covariances

$$P_t^{(ii)} \equiv P_t(\theta_i), \quad i = 1,\ldots,N. \qquad (15)$$

3.3. Fusion of Local Kalman Estimates

Suppose we have N LKEs (11) or (14) with the corresponding LECs (12) or (15). Then the *overall* suboptimal linear estimate of state is given by

$$\hat{x}^{sub} = \sum_{i=1}^{N} c_t^{(i)}\hat{x}_t^{(i)}, \qquad \sum_{i=1}^{N} c_t^{(i)} = I_n. \qquad (16)$$

where I_n is the $n \times n$ init matrix, and $c_t^{(1)},\ldots,c_t^{(N)}$ are $n \times n$ weight matrices determined from the mean square criterion

$$E\left\|x - \hat{x}^{sub}\right\|^2 = E\left(\left\|x_t - \sum_{i=1}^{N} c_t^{(i)}\hat{x}_t^{(i)}\right\|^2\right) \to \min_{c_t^{(i)}} \quad (17)$$

The following theorem completely defines the estimate $\hat{x}_t^{sub}$ and its error covariance

$$P_t^{sub} = \mathrm{cov}\left(e_t^{sub}, e_t^{sub}\right), \quad e_t^{sub} = x_t - \hat{x}_t^{sub}. \qquad (18)$$

Theorem[1]. *Let* $\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}$ *be the local Kalman estimates of an unknown state* x_t. *Then the weight matrices* $c_t^{(1)},\ldots,c_t^{(N)}$ *are given by*

$$\sum_{i=1}^{N} c_t^{(i)}\left[P_t^{(ij)} - P_t^{(iN)}\right] = 0, \quad \sum_{i=1}^{N} c_t^{(i)} = I_n,$$
$$j = 1,\ldots,N-1, \qquad \sum_{i=1}^{N} c_t^{(i)} = I_n. \qquad (19)$$

Corollary 1. If $\hat{x}_t^{(1)},\ldots,\hat{x}_t^{(N)}$ *are unbiased estimates then the suboptimal estimate* $\hat{x}_t^{sub}$ *in (16) is unbiased.*

Corollary 2. The error covariance P_t^{sub} *is given by*

$$P_t^{sub} = \sum_{i,j=1}^{N} c_t^{(i)}P_t^{(ij)}\left(c_t^{(j)}\right)^T. \qquad (20)$$

In the particular case at $N=2$, the formulas (16), (19) are reduced to the Bar-Shalom-Campo formulas [10]:

$$\hat{x}_t^{sub} = c_t^{(1)}\hat{x}_t^{(1)} + c_t^{(2)}\hat{x}_t^{(2)},$$

$$c_t^{(1)} = \left[P_t^{(22)} - P_t^{(21)}\right]\left[P_t^{(11)} + P_t^{(22)} - P_t^{(12)} - P_t^{(21)}\right]^{-1}, \qquad (21)$$

$$c_t^{(2)} = \left[P_t^{(11)} - P_t^{(12)}\right]\left[P_t^{(11)} + P_t^{(22)} - P_t^{(12)} - P_t^{(21)}\right]^{-1}.$$

If these two estimates are uncorrelated, i.e., $P^{(12)} = 0$, then the formulas (11) give the Millman formulas [4]:

$$c_t^{(1)} = P_t^{(22)}\left[P_t^{(11)} + P_t^{(22)}\right]^{-1},$$

$$c_t^{(2)} = P_t^{(11)}\left[P_t^{(11)} + P_t^{(22)}\right]^{-1}.$$

As we see the unknown weights $c_t^{(i)}$ are satisfied to the linear algebraic equations (19) with coefficients depending on the LECs $P_t^{(ij)}, i, j = 1,\ldots,N.$ Usually in various estimation problems the LECs $P_t^{(ii)}, i = 1, \ldots, N$ are known. For instance, in linear filtering problems 3.1 and 3.2 the LECs $P_t^{(ii)}$ are described by the Kalman filter equations [see (12) or (15)]. However the cross-covariances $P_t^{(ij)},\ i \neq j$, are usually unknown and it should be determined in each specific filtering problem.

3.4. Equation for Cross-Covariance in Filtering Problem with Multi-Sensors Environment

In the filtering problem (1), (3), (4) the cross-covariance $P_t^{(ij)}$, where $i \neq j$, satisfies to the following differential equation:

$$
\begin{aligned}
\dot{P}_t^{(ij)} &= \left(F_t - P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}} H_t^{(i)}\right) P_t^{(ij)} \\
&+ P_t^{(ij)} \left(F_t - P_t^{(jj)} H_t^{(j)^T} R_t^{(j)^T} H_t^{(j)}\right)^T \\
&+ G_t Q_t G_t^T, \quad P_0^{(ij)} = P_0; \ \ i \neq j.
\end{aligned} \tag{22}
$$

where $P_t^{(ii)}, i = 1, \ldots, N$ represent the LECs (12).

Thus the LKEs (11), LECs (12), and equations (16), (19), (22) completely define *the two-steps suboptimal filter (TSSF)* for dynamic system with multi-sensors environment (1), (3), (4).

3.5. Equation for Cross-Covariance in Adaptive Filtering Problem

In the adaptive filtering problem (1),(2),(6) the cross-covariance $P_t^{(ij)},\ i \neq j$ also satisfies to the equation (22) with matrices

$$H_t^{(i)} \equiv H_t^{(i)}(\theta_i), \quad R_t^{(i)} \equiv R_t^{(i)}(\theta_i). \tag{23}$$

and LECs $P_t^{(ii)}$ (15).

Thus the LKEs (14), LECs (15), and equations (16), (19), (22) completely define TSSF for dynamic system with observation uncertainties (1), (2), (6).

Remark 1. The LKEs are separated, i.e., each local estimate $\hat{x}_t^{(i)}$ is found independently of other estimates. Therefore, the LKEs can be calculated in parallel. The proposed TSSF is also robust, since it can be corrected even if one of the parallel local estimates $\hat{x}_t^{(i)}$ diverges. In this case, the corresponding weight matrix $c_t^{(i)}$ in (16) will tend to zero, thereby indicating that the diverging local estimate $\hat{x}_t^{(i)}$ will be discarded in the weighting sum.

Remark 2. We may note, that the all covariances $P_t^{(ij)}$, and the weights $c_t^{(i)}$ may be pre-computed, since they do not depend on the measurements Y_t, but only on the noises statistics $Q_t, R_t^{(i)}$, and the system matrices $F_t, G_t, H_t^{(i)}$, which are the part of system and measurement model (1), (2). Thus, once the measurement schedule has been settled, the real-time implementation of the TSSF requires only the computation of the LKEs and the final suboptimal estimate $\hat{x}_t^{sub}$.

Remark 3. In case of one sensor ($N = 1$) and completely known parameter $\theta = \theta^*$, the TSSF coincides with KF.

Remark 4. The TSSF can also be used for distributed parallel data fusion system, provided better communication system and individual measuring device with processing capabilities.

4. ESTIMATION OF DAMPER HARMONIC OSCILLATOR MOTION

In this section, we verify TSSFs for model of the harmonic oscillator [10]

$$\dot{x}_t = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\alpha \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t, \quad t \geq 0. \tag{24}$$

where $x_t = \begin{bmatrix} x_{1,t} & x_{2,t} \end{bmatrix}^T$ and $x_{1,t}$ is position, and $x_{2,t}$ is velocity, $v_t \sim (0, q)$, $x_0 \sim N(\bar{x}_0, P_0)$.

We consider two observation models:

4.1. Two sensors for measuring a position

In the first observation model position $x_{1,t}$ is only measured by two different sensors which are given

$$y_t^{(1)} = x_{1,t} + w_t^{(1)}, \quad y_t^{(2)} = x_{2,t} + w_t^{(2)}. \tag{25}$$

where $w_t^{(1)} \sim (0, r_1)$ and $w_t^{(2)} \sim (0, r_2)$ are uncorrelated white noises.

For this case two filters for the system model (24), (25) are considered:

A). The KF based on the *overall* observation model (3),

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad w_t = \begin{bmatrix} w_t^{(1)} \\ w_t^{(2)} \end{bmatrix}.$$

B). The TSSF based on the Bar-Shalom-Campo formulas (21),

$$\hat{x}_t^{sub} = c_t^{(1)}\hat{x}_t^{(1)} + c_t^{(2)}\hat{x}_t^{(2)}. \quad (26)$$

where $\hat{x}_t^{(1)}$ and $\hat{x}_t^{(2)}$ are the LKEs (11) based on the individual sensors $y_t^{(1)}$ and $y_t^{(2)}$, respectively.

To study the behavior of the KF and TSSF error covariances, let $\omega_n^2 = 0.64$, $\alpha = 0.16$, $q = 1$, $r_1 = 0.02$, $r_2 = 0.01$, and $P_0 = diag[2 \quad 1]$. The point of interest is the mean square errors (MSE) in the estimate of state components,

$$P_{ii,t}^{KF} = E\left[\left(e_{i,t}^{KF}\right)^2\right], \quad e_{i,t}^{KF} = x_{i,t} - \hat{x}_{i,t}^{KF}, \quad i = 1,2,$$
$$P_{ii,t}^{sub} = E\left[\left(e_{i,t}^{sub}\right)^2\right], \quad e_{i,t}^{sub} = x_{i,t} - \hat{x}_{i,t}^{sub}, \quad i = 1,2. \quad (27)$$

where $e_t^{KF} = \left[e_{1,t}^{KF} \quad e_{2,t}^{KF}\right]^T$ is the estimation error of the state components under consideration at time **t** with optimal KF, and similarly for TSSF (26). These are the quantities shown in Fig.1. Fig.1 shows the comparison of MSEs (27) for KF and TSSF.

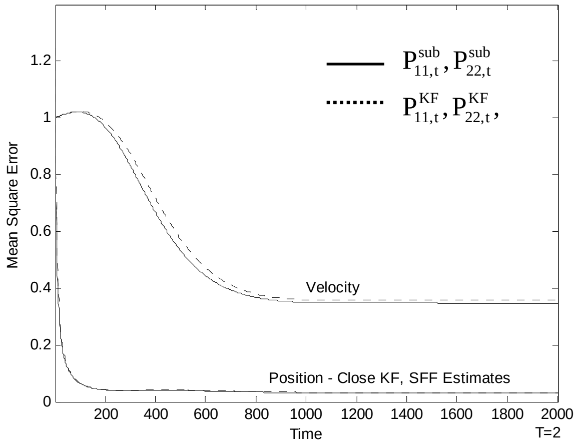

Fig. 1 MSE analysis of KF and TSSF for position and velocity

As it is seen from Fig.1, the differences between $P_{ii,t}^{KF}$ and $P_{ii,t}^{sub}$ are negligible, especially for steady-state regime. This means that for our example the application of TSSF can produce good results in real-time processing requirements. MSEs can be further minimized by increasing the number of sensors $N > 2$.

4.2. Joint Detection and Estimation

In this case the observation equation is written as

$$y_t = \theta x_{1,t} + w_t = [\theta \quad 0]x_t + w_t. \quad (28)$$

and, $w_t \sim (0, r)$ is white noise. where the unknown parameter θ takes only two values, i.e.,

$$\theta = \begin{cases} \theta_1 = 1, & p_1 = \Pr(\theta = \theta_1) \\ \theta_2 = 0, & p_1 = \Pr(\theta = \theta_2) \end{cases}. \quad (29)$$

This represents the observation model which takes two sensor modes with $\theta_1 = 1$ (signal-present) and $\theta_2 = 0$ (signal-absent). We describe here the results of simulations of two filters: the optimal LKF (7), (8) and TSSF (26), in which $\hat{x}_t^{(1)}$ and $\hat{x}_t^{(2)}$ are the LKEs matched to the linear system (24), (28) at $\theta_1 = 1$ and $\theta_2 = 0$, respectively. Note that the TSSF's weights $c_t^{(i)}$ in (26) represent function on time as distinct from the LKF's weights $\tilde{c}_t^{(i)}$ in (8), which depend on present observations y_t. The values of parameters are $\omega_n^2 = 0.64$, $\alpha = 0.16$, $q = 1$, $r = 0.01$ and $P_0 = diag[2 \quad 1]$.

Two cases were considered: in the first case, $\theta_1 = 1$ is the true parameter value; for the second case, $\theta_2 = 0$ is the true parameter value. The Fig.2 and 3 present the mean square errors (MSE) of position $x_{1,t}$ and velocity $x_{2,t}$ for the first case, respectively. Such time histories are perfect analogy to the second case. The analysis of the results shows that the TSSF yields good accuracy. It also provides the best balance between computational efficiency and desired estimation accuracy.

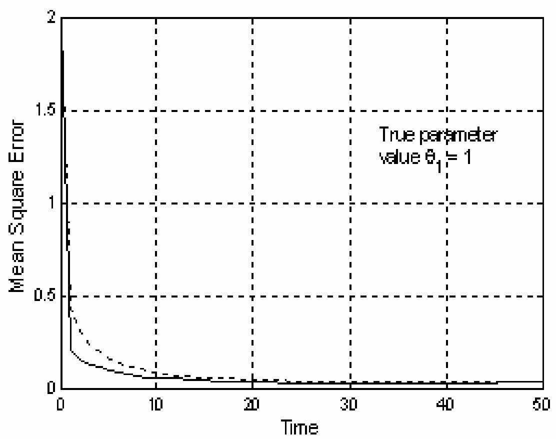

Fig. 2 Illustration of the optimal and suboptimal MSE in position: MSE of optimal LKF (*solid line*), MSE of TSSF (*dotted line*).

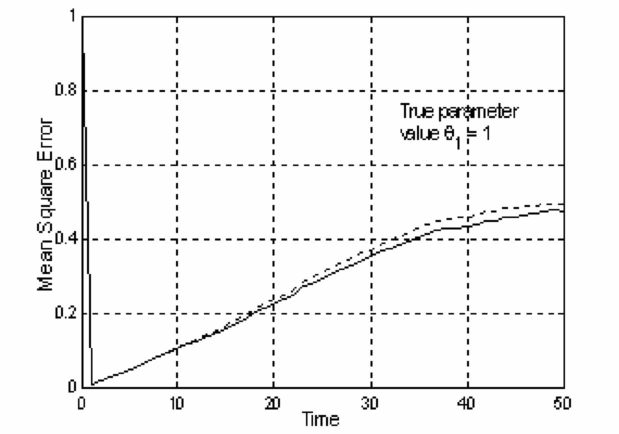

Fig. 3 Illustration of the optimal and suboptimal MSE in velocity: MSE of optimal LKF (*solid line*), MSE of TSSF (*dotted line*).

5. CONCLUSION

In this paper, we have designed the new two-step suboptimal filters for linear dynamic systems with multi-sensor environment and observation uncertainties. The filters represent linear combination of local Kalman filters. Each local filter is fused by the minimum mean square criterion. The new filters have parallel structure and are very suitable for parallel processing. Simulation results demonstrated the high accuracy of the design filters.

The proposed filter can be used in the various areas: industrial, military, space, communication, target tracking, inertial navigation and others [6], [7], [8].

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