A Sliding Surface Design for Linear Systems with Mismatched Uncertainties based on Linear Matrix Inequality

Seung Ho Jang* and Sang Woo Kim**

*Digital Business Division, SHI Co., Ltd., 493, Banweol-Ri, Taean-Eup, Hwasung-City, Gyeounggi-Do, 445-973, Korea
(Tel: +82-31-229-1303; Fax: +82-31-229-1420; Email: sh71.jang@samsung.com)
**Electrical and Computer Engineering Division, POSTECH, Pohang, 790-784, Korea
(Tel: +81-54-279-2237; Fax: +81-54-279-2903; Email: swkim@postech.ac.kr)

Abstract: Sliding mode control (SMC) is an effective method of controlling systems with uncertainties which satisfy the so-called matching condition. However, how to effectively handle mismatched uncertainties of systems is still an ongoing research issue in SMC. Several methods have been proposed to design a stable sliding surface even if mismatched uncertainties exist in a system. Especially, it is presented that robustness and efficiency of SMC for linear systems with mismatched uncertainties can be improved by reducing mismatched uncertainties in the reduced-order system. The reduction method needs a new sliding surface with an additional component based on Lyapunov redesign technique. In this paper, a stable sliding surface which contains additional component to reduce the influence of mismatched uncertainties, is introduced. It is designed by using linear matrix inequalities that guarantees the stability of the system. A numerical example demonstrates the validity of the proposed scheme.

Keywords: sliding mode control, mismatched uncertainty, linear matrix inequality, robust control

1. Introduction

SMC is an effective method of controlling uncertain systems. Since the system trajectories are constrained on the predetermined sliding surfaces in sliding mode, SMC has a good performance. The system behavior is insensitive to the internal parameter variations and external disturbances, if uncertainties and/or disturbances of the system satisfy the so-called matching condition in this sliding mode. It is possible since uncertainties of the system satisfying the matching condition are completely nullified in sliding mode. Although the system is in sliding mode, the effect of mismatched uncertainties of the system remains so that adaptation method, robust method and backstepping method have been used to handle the system with mismatched uncertainties in SMC [1] [3] [4] [5]. Recently, it is shown that robustness and efficiency of SMC for linear systems with mismatched uncertainties can be improved via the reduction of mismatched uncertainties [1].

In sliding mode, an original uncertain system can be changed to the equivalent reduced-order system [7]. Although matched uncertainties are nullified, mismatched uncertainties still remain in this reduced-order system. Here, mismatched uncertainties can be divided into the matched part, which is in the range space of the input matrix, and the rest part in the reduced-order system [1]. Then, a sliding surface should be designed to nullify the matched part and to stabilize with respect to the rest part.

In this paper, we show a stable sliding surface, which contains an additional component to nullify the effect of the match part, can be designed via LMIs if the bound of the rest part satisfies the structured norm bound. A numerical example is given to explain the proposed method and to demonstrate its validity.

This paper is organized as follows: Section 2 briefly introduces uncertain systems considered in this paper. In Section 3, the sufficient condition of stable sliding surface is presented in terms of LMI. Section 4 introduces the reduction method of mismatched uncertainties as well as presents the sufficient condition of new sliding surface with an additional component. In Section 5, sliding mode control law to guarantee a reaching condition is presented. In Section 6, a numerical example is given to illustrate design procedure as well as its validity. Finally, Section 7 serves as a conclusion.

2. Problem statement

Consider the following linear uncertain system:

$$\dot{x}(t) = (A + \Delta A)x(t) + B\left[(I + \Delta B)u(t) + \Delta f(t,x)\right], \quad (1)$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^m$ is the control input and Δ denotes uncertainties. A and B are known constant matrices with appropriate dimensions. Generally, there exists a invertible transformation $z = Tx$ such that transforms the system dynamics Eq. (1) to its regular form. Therefore, without loss of generality, the system is described in the regular form such as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \left\{(I + \Delta B_2)u + \Delta f_m\right\}, \quad (2)$$

where $x_1 \in \mathcal{R}^{n-m}$ and $x_2 \in \mathcal{R}^m$ are the state vectors, $u \in \mathcal{R}^m$ is control input and $\Delta f_m \in \mathcal{R}^m$ represent the lumped matching uncertainties. ΔA_{11} and ΔA_{12} become mismatched uncertainties. A_{11}, A_{12}, A_{21}, A_{22} and B_2 are known constant matrices with appropriate dimensions. Furthermore, bounds of Δf_m and ΔB_2 are assumed to be known.

Assumption 1: Uncertainties Δf_m,ΔB_2, ΔA_{11} and ΔA_{12} are bounded in Euclidean norm by known functions as, $\forall (x,t) \in \mathcal{R}^n \times \mathcal{R}^+$, $||\Delta f_m|| \le \rho_m(x)$, $||\Delta B_2|| \le 1 - \epsilon_b$ and $||\Delta A_{11}\ \Delta A_{12}|| \le \eta$ where ϵ_b and η are positive constants.

Assumption 2: The pair (A, B)

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix},$$

is controllable, and the matrix B_2 has full rank.
Assumption 2 is necessary for the existence of the equivalent control in Section 6.

3. Design of sliding surface using LMI

A linear sliding surface is usually defined as

$$\sigma(x) = Sx = 0,$$

or without loss of generality

$$\sigma(x) = \bar{K}x = \begin{bmatrix} K & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0, \tag{3}$$

where $S \in \mathcal{R}^{m \times n}$ and $K \in \mathcal{R}^{m \times (n-m)}$. In sliding mode, $\sigma(x) = 0$,

$$x_2 = -Kx_1, \tag{4}$$

and

$$\dot{x}_1 = (A_{11} + \Delta A_{11})x_1 - (A_{12} + \Delta A_{12})Kx_1, \tag{5}$$

which is the reduced-order dynamics. If the uncertainties, ΔA_{11} and ΔA_{12}, are admissibly norm-bounded and structure as following:
Assumption 3: Assume that

$$\Delta A_{11} = D_r F_r E_{r1},$$
$$\Delta A_{12} = D_r F_r E_{r2},$$

where D_r, E_{r1} and E_{r2} are known real constant matrices of appropriate dimensions, and F_r is an unknown matrix function with Lebesque-measurable elements and satisfies $F_r^T F_r \leq I$, in which I is the identity matrix.
Under the above condition, the sliding surface Eq. (3), which stabilizes the reduced-order system Eq. (5) in the presence of mismatched uncertainties, can be designed in terms of LMI [3]. In order to show the proof, we need to recall the following matrix inequality.
Lemma 1 [2]: Given constant matrices D and E and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S + DFE + E^T F^T D^T < 0,$$

where F satisfies $F^T F \leq R$, if and only if for some $\gamma > 0$

$$S + [\gamma^{-1}E^T \quad \gamma D] \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \gamma^{-1}E \\ \gamma D^T \end{bmatrix} < 0.$$

The main result on the global asymptotic stability of the reduced-order system with mismatched uncertainties is summarized in the following theorem.

Theorem 1: If there exist a symmetric and positive definite matrix P_r, some matrix K and some positive γ such that the following LMI are satisfied, then the reduced-order system Eq. (5) is asymptotically stabilizable via the sliding mode surface Eq. (3):

$$\begin{bmatrix} \Psi_r & * & \\ E_{r1}Q_r - E_{r2}M_r & -\gamma I & * \\ D_r^T & 0 & -\gamma^{-1}I \end{bmatrix} < 0, \tag{6}$$

where $\Psi_r = Q_r A_{11}^T + A_{11}Q_r - M_r^T A_{12}^T - A_{12}M_r$, $Q_r = P_r^{-1}$ and $M_r = KP_r^{-1}$, where * denotes the transposed elements in the symmetric positions.

Proof: Consider Lyapunov function candidate

$$V = x_1^T P_r x_1, \tag{7}$$

where P_r is a time-invariant and positive definite matrix. Clearly, V is positive definite and radially unbounded. The time derivative of V is

$$\dot{V} = \dot{x}_1^T P_r x_1 + x_1^T P_r \dot{x}_1. \tag{8}$$

By substituting Eq. (5) into Eq. (8), we obtain as follow

$$\begin{aligned} \dot{V} = x_1^T \Phi_r x_1 &+ x_1^T (\Delta A_{11} - \Delta A_{12}K)^T P_r x_1 \\ &+ x_1^T P_r (\Delta A_{11} - \Delta A_{12}K) x_1, \end{aligned} \tag{9}$$

where $\Phi_r = A_{11}^T P_r + P_r A_{11} - K^T A_{12}^T P_r - P_r A_{12} K$. If the right hand side of Eq. (9) is negative definite uniformly for all x_1 and $t \geq 0$ except at $x_1 = 0$ then the reduced-order dynamics Eq. (5) is asymptotically stable about its zero equilibrium. Therefore, the following inequality is valid.

$$\Phi_r + (\Delta A_{11} - \Delta A_{12}K)^T P_r + P_r(\Delta A_{11} - \Delta A_{12}K) < 0. \tag{10}$$

Then, applying Assumption 3 to (10) yields

$$\Phi_r + \Sigma_r^T F_r^T D_r^T P + P D_r F_r \Sigma_r < 0, \tag{11}$$

where $\Sigma_r = E_{r1} - E_{r2}K$. According to Lemma 1, the matrix inequality Eq. (11) holds for all F_r satisfying $F_r^T F_r \leq I$ if only if there exists a constant $\gamma^{1/2} > 0$ such that

$$\Phi_r + \begin{bmatrix} \gamma^{-1/2}\Sigma_r \\ \gamma^{1/2}(P_r D_r)^T \end{bmatrix}^T \times \begin{bmatrix} \gamma^{-1/2}\Sigma_r \\ \gamma^{1/2}(P_r D_r)^T \end{bmatrix} =$$

$$\Phi_r + \begin{bmatrix} \Sigma_r \\ D^T P_r \end{bmatrix}^T \begin{bmatrix} \gamma^{-1}I & 0 \\ 0 & \gamma I \end{bmatrix} \begin{bmatrix} \Sigma_r \\ D^T P_r \end{bmatrix} < 0. \tag{12}$$

Applying Schur complement to Eq. (12) result in

$$\begin{bmatrix} \Phi_r & * & * \\ \Sigma_r & -\gamma I & * \\ D_r^T P_r & 0 & -\gamma^{-1}I \end{bmatrix} < 0. \tag{13}$$

Define the following transformation matrix as:

$$\begin{bmatrix} P_r^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{14}$$

and take a congruence transformation. This yields

$$\begin{bmatrix} P_r^{-1}\Phi_r P_r^{-1} & * & * \\ \Sigma_r P_r^{-1} & -\gamma I & * \\ D_r^T & 0 & -\gamma^{-1}I \end{bmatrix} < 0. \tag{15}$$

Denoting $Q_r = P_r^{-1}$ and $M_r = KP_r^{-1}$ yields the LMI Eq. (6). This completes the proof of the theorem. ■

4. Design of new sliding surface

In this section, a reduced-order dynamics associated with a new sliding surface with an additional component is presented. Additionally, the sufficient condition of new stable sliding surface in terms of LMI is derived. Consider the following new sliding surface:

$$\sigma(x) = Kx_1 + v + x_2. \tag{16}$$

In sliding mode, the reduced-order dynamics will be

$$\dot{x}_1 = (A_{11} + \Delta A_{11})x_1 - (A_{12} + \Delta A_{12})\{Kx_1 + v\}. \tag{17}$$

First of all, mismatched uncertainties ΔA_{11} and ΔA_{12} should be divided into the matched part, which is in the range space of A_{12}, and the rest part. That is, D_r should be divide into the projection on the range space of A_{12} and the rest such as

$$D_r = A_{12}T_p + \bar{D}_r.$$

Therefore, mismatched uncertainties ΔA_{11} and ΔA_{12} are rewritten as follows

$$\Delta A_{11} = A_{12}\bar{\Delta}A_{11} + \Delta A_{ru1}, \tag{18}$$

$$\Delta A_{12} = A_{12}\bar{\Delta}A_{12} + \Delta A_{ru2}, \tag{19}$$

where $\bar{\Delta}A_{11} = T_pF_rE_{r1}$, $\bar{\Delta}A_{12} = T_pF_rE_{r2}$, $\Delta A_{ru1} = \bar{D}_rF_rE_{r1}$ and $\Delta A_{ru2} = \bar{D}_rF_rE_{r2}$. Then, the reduced-order dynamics Eq. (17) becomes

$$\begin{aligned}\dot{x}_1 = A_{11}x_1 + A_{12}\left[-(I+\bar{\Delta}A_{12})\{Kx_1+v\}\right.\\ \left.+\Delta f_{rm}\right] + \Delta f_{ru},\end{aligned} \tag{20}$$

where

$$\begin{aligned}\Delta f_{rm} &= \bar{\Delta}A_{11}x_1,\\ \Delta f_{ru} &= \Delta A_{ru1}x_1 - \Delta A_{ru2}\{Kx_1+v\}.\end{aligned}$$

Suppose, as usual, that the uncertainties are admissibly norm-bounded.

Assumption 4: There exists $\epsilon > 0$ such that $||\bar{\Delta}A_{12}|| \leq 1-\epsilon$. And, uncertainty Δf_{rm} is norm-bounded as $||\Delta f_{rm}|| \leq \rho_{rm}||x_1||$.

We define a candidate of Lyapunov function $V(x_1)$ mapping from $\mathcal{R}^{n-m}$ to $\mathcal{R}$ such as

$$V(x_1) = x_1^TPx_1, \tag{21}$$

where $P \in \mathcal{R}^{(n-m)\times(n-m)}$ is a positive definite matrix. Furthermore, suppose the following additional component

$$v = \frac{\bar{\rho}_{rm}^2}{2\epsilon\zeta}A_{12}^TPx_1, \tag{22}$$

where ζ is a positive scalar and $\bar{\rho}_{rm} = \rho_{rm} + (1-\epsilon)||K||$. For simplicity of notation, a new function is defined as follows

$$w = 2A_{12}^TPx_1. \tag{23}$$

Then, the global asymptotic stability of the uncertain reduced-order system Eq. (20) is summarized in the following theorem.

Theorem 2: Assume that Assumption 4 is satisfied and the additional surface component v is given such as Eq. (22). If there exist a symmetric and positive definite matrix P, some matrix K and some positive γ and ζ such that the following LMIs are satisfied, then the uncertain reduced-order Eq. (20) is asymptotically stabilizable associated with the new sliding mode surface Eq. (16):

$$\begin{bmatrix} \Psi & * & * & * \\ \Omega & -\gamma I & * & * \\ \bar{D}_r^T & 0 & -\gamma^{-1}I & * \\ Q & 0 & 0 & -\zeta^{-1}I \end{bmatrix} < 0, \tag{24}$$

where $\Psi = QA_{11}^T + A_{11}^TQ - M^TA_{12}^T - A_{12}M$, $\Omega = E_{r1}Q - E_{r2}M - \frac{\bar{\rho}_{rm}^2}{4\epsilon}\zeta^{-1}E_{r2}A_{12}^T$, $Q = P^{-1}$ and $M = KP^{-1}$, and * denotes the transposed elements in the symmetric positions.

Proof: Consider the time derivative of the Lyapunov function Eq. (21).

$$\begin{aligned}\dot{V}(x_1) = x_1^T(A_{11} - A_{12}K)^TPx_1 - x_1^TP(A_{11} - A_{12}K)x_1\\ +2x_1^TPA_{12}\left\{-(I+\bar{\Delta}A_{12})v - \bar{\Delta}A_{12}Kx_1\right.\\ \left.+\Delta f_{rm}\right\} + 2x_1^TP\Delta f_{ru}.\end{aligned} \tag{25}$$

Using Assumption 4 and Eq. (22), the third term of Eq. (25) satisfies the following inequality

$$\begin{aligned}&2x_1^TPA_{12}\left\{-(I+\bar{\Delta}A_{12})v - \bar{\Delta}A_{12}Kx_1 + \Delta f_{rm}\right\}\\ &\quad\leq -w^T(I+\bar{\Delta}A_{12})v + ||\bar{\Delta}A_{12}K||||w||||x_1||\\ &\qquad\qquad +\rho_{rm}||w||||x_1||\\ &\quad\leq -\frac{\epsilon\bar{\rho}_{rm}^2}{4\epsilon\zeta}w^Tw + \{(1-\epsilon)||K|| + \rho_{rm}\}||w||||x_1||\\ &\quad= -\begin{bmatrix}\frac{\bar{\rho}_{rm}}{2}||w||\\ ||x_1||\end{bmatrix}^T\begin{bmatrix}\zeta^{-1} & -1\\ -1 & \zeta\end{bmatrix}\begin{bmatrix}\frac{\bar{\rho}_{rm}}{2}||w||\\ ||x_1||\end{bmatrix}\\ &\qquad\qquad +\zeta||x_1||^2\\ &\quad\leq \zeta||x_1||^2 = \zeta x_1^Tx_1.\end{aligned} \tag{26}$$

Therefore, we obtain the following condition from Eq. (25) and Eq. (26)

$$\begin{aligned}\dot{V}(x_1) \leq x_1^T(A_{11} - A_{12}K)^TPx_1 + x_1^TP(A_{11} - A_{12}K)x_1\\ +\zeta x_1^Tx_1 + 2x_1^TP\Delta f_{ru}\\ = x_1^T\Phi x_1 + \zeta x_1^Tx_1 + 2x_1^TP\bar{D}_rF_rE_{r1}x_1\\ -2x_1^TP\bar{D}_rF_rE_{r2}(Kx_1+v),\end{aligned} \tag{27}$$

where $\Phi = A_{11}^TP + PA_{11} - K^TA_{12}^TP - PA_{12}K$.

Using Eq. (22),

$$\begin{aligned}2x_1^TP\bar{D}_rF_rE_{r1}x_1 - 2x_1^TP\bar{D}_rE_{r2}(K + \frac{\bar{\rho}_{rm}^2}{2\epsilon\zeta}A_{12}^TP)x_1\\ = x_1^TP\bar{D}_rF_r\Sigma x_1 + x_1^T\Sigma^TF_r^T\bar{D}_r^TPx_1,\end{aligned} \tag{28}$$

where $\Sigma = E_{r1} - E_{r2}K - \frac{\bar{\rho}_{rm}^2}{2\epsilon}\zeta^{-1}E_{r2}A_{12}^TP$. Therefore, we obtain the following condition from Eq. (27) and Eq. (28)

$$\begin{aligned}\dot{V}(x_1) \leq x_1^T\Phi x_1 + \zeta x_1^Tx_1 + x_1^TP\bar{D}_rF_r\Sigma x_1\\ +x_1^T\Sigma^TF_r^T\bar{D}_r^TPx_1.\end{aligned} \tag{29}$$

If the right hand side of Eq. (29) is negative definite uniformly for all x_1 and $t \geq 0$ except at $x_1 = 0$ then the reduced-order dynamics Eq. (20) is asymptotically stable about its zero equilibrium. Therefore, assume that the following inequality is valid.

$$\Phi + \zeta I + P\bar{D}_r F_r \Sigma + \Sigma^T F_r^T \bar{D}_r^T P < 0. \qquad (30)$$

The conversion of Eq. (30) to LMI Eq. (24) follows the procedure in Theorem 1. ■

5. Sliding mode control

In the previous section, when the uncertain system Eq. (2) was in the sliding mode, the sliding mode surface was designed to guarantee the asymptotic stability of the reduced-order system in terms of LMI. Next, we need to find feedback control law u to drive state trajectories of the system onto the sliding surface. This means that the control law is designed to guarantee the existence of sliding mode or the reaching condition. Before proceeding, for notational simplicity, let the proposed sliding surface be redefined as

$$\begin{aligned}\sigma(x) &= Kx_1 + v + x_2 = Kx_1 + \frac{\bar{\rho}_{rm}^2}{4\epsilon\zeta} w + x_2 \\ &= (K + \frac{\bar{\rho}_{rm}^2}{2\epsilon\zeta} A_{12}^T P)x_1 + x_2 = K_T x_1 + x_2, \qquad (31)\end{aligned}$$

where $K_T = K + \frac{\bar{\rho}_{rm}^2}{2\epsilon\zeta} A_{12}^T P$. The feedback control law satisfying reaching condition is summarized in the following theorem.

Theorem 3: For the system Eq. (2) satisfying Assumption 1 and 2, the following control law is considered:

$$u = \begin{cases} u_{eq} & if\ \sigma(x) = 0, \\ u_{eq} - \rho(x)\frac{B_2\sigma(x)}{||B_2\sigma(x)||} & if\ \sigma(x) \neq 0, \end{cases} \qquad (32)$$

where

$$u_{eq} = -B_2^{-1}\begin{bmatrix} K_T \\ I \end{bmatrix}\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x,$$

$$\rho(x) > \frac{(1-\epsilon_b)||u_{eq}|| + \eta||B_2^{-1}K_T||||x|| + \rho_m(x)}{\epsilon_b}.$$

Then, the reaching condition of the sliding surface Eq. (31) is satisfied.

Proof: See reference [1]. ■

6. Numerical example

In this section, we give the results of simulation study. Consider the following uncertain system which satisfies Assumption 1, Assumption 2 and Assumption 3:

$$\dot{x} = \begin{bmatrix} a_{11}+\Delta a_{11} & a_{12}+\Delta a_{12} & a_{13}+\Delta a_{13} \\ a_{21}+\Delta a_{21} & a_{22}+\Delta a_{22} & a_{23}+\Delta a_{23} \\ a_{31}+\Delta a_{31} & a_{32}+\Delta a_{32} & a_{33}+\Delta a_{33} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\{(I+\Delta B)u + \Delta f\}, \qquad (33)$$

where

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 & 0.2 \\ 1.2 & 2.6 & 0.8 \\ -0.4 & 1.7 & 0.6 \end{bmatrix},$$

$$\begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix} = \begin{bmatrix} 0.036 & -0.03 \\ 0.161 & -0.12 \end{bmatrix}\begin{bmatrix} \sin\pi t + 1 & -1 \\ -\cos\pi t & \cos\pi t \end{bmatrix},$$

$$\begin{bmatrix} \Delta a_{13} \\ \Delta a_{23} \end{bmatrix} = \begin{bmatrix} 0.036 & -0.03 \\ 0.161 & -0.012 \end{bmatrix}\begin{bmatrix} \sin\pi t \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} \Delta a_{31} \\ \Delta a_{32} \\ \Delta a_{33} \end{bmatrix} = \begin{bmatrix} 0.03 + 0.14\sin 2\pi t \\ 0 \\ 0.19\cos 1.3\pi t \end{bmatrix},$$

$$\Delta B = 0.1\cos 1.5\pi t,\ \Delta f = 0.7\sin 2\pi t + 0.4.$$

In sliding mode, the reduced-order dynamics of Eq. (33) can be described as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 1.2 & 2.6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} x_3 + \begin{bmatrix} \Delta a_{13} \\ \Delta a_{23} \end{bmatrix} x_3. \qquad (34)$$

In the reduced-order dynamics, mismatched uncertainties can be decomposed such as

$$\begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix} = D_r F_r E_{r1},$$

$$\begin{bmatrix} \Delta a_{13} \\ \Delta a_{23} \end{bmatrix} = D_r F_r E_{r2},$$

where

$$D_r = \begin{bmatrix} 0.036 & -0.03 \\ 0.161 & -0.12 \end{bmatrix},\ E_{r1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},$$

$$F_r = \begin{bmatrix} \sin\pi t & -1 \\ 0 & \cos\pi t \end{bmatrix},\ E_{r2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Also, D_r should be divided into the projection on $\begin{bmatrix} 0.2 & 0.8 \end{bmatrix}^T$ and the rest as follows

$$D_r = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}[0.2 \quad -0.15] + \begin{bmatrix} -0.004 & 0 \\ 0.001 & 0 \end{bmatrix}.$$

The sliding surface should be designed by Theorem 2 under the given conditions that:

$$P = \begin{bmatrix} 3.1514 & 0.0315 \\ 0.0315 & 3.9924 \end{bmatrix},\ K = \begin{bmatrix} 3.0595 \\ 13.1628 \end{bmatrix}^T, \zeta = 1.1643.$$

In Figure 1, we compared with the simulation results of the sliding mode controllers which are associated with the sliding surfaces designed by Theorem 1 and Theorem 2, respectively. simulation results show that if mismatched uncertainties are split, the distortion, which is occurred by mismatched uncertainties,is more considerably decreased. This clearly demonstrates the robustness improvement of reduction of mismatched uncertainties over no-reduction of it. This result means that the robustness of the proposed method is better than no reduction method.

7. Conclusion

A new design method of the sliding surface for linear systems with mismatched uncertainties is proposed. This scheme enables us to handle mismatched uncertainties by means of offering compensation for the matched part which is in the range space of the input channel of the reduced-order systems. Additionally, the stabilizing criteria of sliding surface was formulated in terms of LMI. Furthermore, effectiveness of the proposed method is demonstrated by a numerical example.

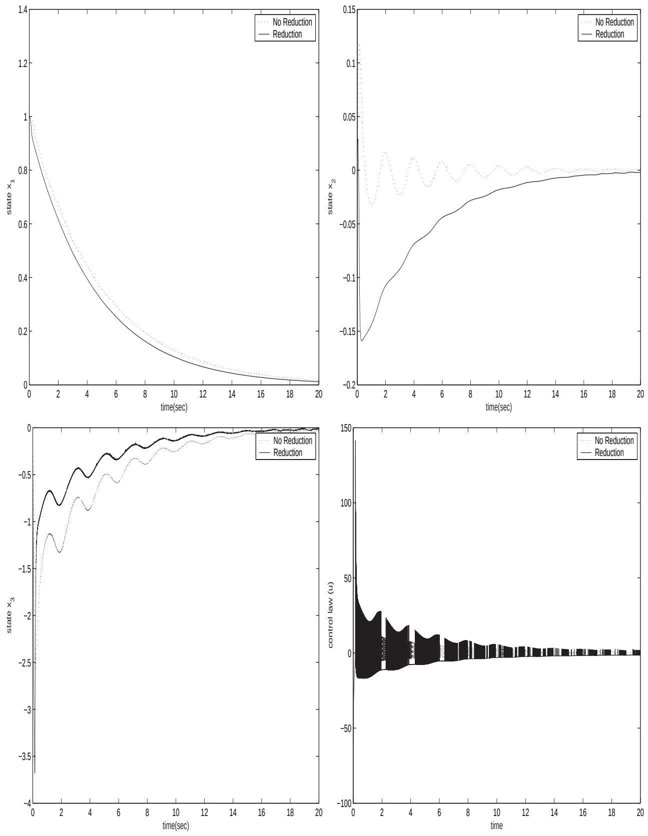

Fig. 1. Simulation results of mismatched uncertainty reduction and noreduction

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