A Note on State Estimation Problems for Perspective Linear Systems Corrupted by Noises

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Abstract: Perspective dynamical systems arise in machine vision problems, in which only perspective observation is available. This paper considers the state estimation problem for a rigid body moving in three dimensional spaces using the image data obtained by a CCD camera or some other means. Because the motion of the rigid body and the observed data are generally corrupted by noises, it is necessary to seek a state estimation method to reduce the influence of the noises. In this paper, by means of computer simulations for a simple example, we examine the sensitivity to the noises of the nonlinear observer developed in the recent paper ([1] R. Abdursul, H. Inaba and B. Ghosh, Nonlinear observers for perspective time-invariant linear systems, *Automatica,* vol. 40, Issue 3, pp. 481-490, 2004) and the effectiveness of the Extended Kalman Filter.

Keywords: Perspective system, Nonlinear observer, Luenberger-type observer, Extended Kalman Filter, Machine vision

1. INTRODUCTION[1]

The observation obtained by observing a moving rigid body by a camera is essentially the direction vector of a point observed, called *a perspective observation*, because all the points on a line passing through the center of the camera are projected to a single point of the image plane. And the basic problem in dynamical machine vision is how to determine the position and velocity of a moving body and/or any unknown parameters characterizing the motion and shape of the body from such perspective observation. A perspective dynamical system arises in mathematically describing such a dynamic machine vision problem and has been studied in a number of approaches in the framework of systems theory (See, e.g., [1],[3]-[11]).

Some interesting works recently reported are concerned with nonlinear observers for estimating the unknown state of perspective time-varying linear systems [1], [3]-[11]. In particular, the paper [1] has proposed a nonlinear observer of the Luenberger-type for such a time-invariant system without transforming it into an implicit system as proposed and discussed in the papers [9],[10] and shown that under some reasonable assumptions on the given system the estimation error of the proposed nonlinear observer converges exponentially to zero.

In these previous works it was assumed that the motion equation of the rigid body and its observed data are not corrupted by noises. However in more realistic situations the observed data, even the motion, may be corrupted by noises. A similar situation occurs in computer vision problems. In fact the work [6] (Chiuso*, et a.,* 1995) discussed a problem of computer vision and a nonlinear filter was proposed in order to causally estimate the 3-dimentional shape by integrating noisy visual information over time.

The objective of the present paper is to examine the sensitivity to the noises of the nonlinear observer developed in the recent paper [1] and the effectiveness of the Extended Kalman Filter.

First, in Section 2, a perspective time-varying linear system considered in this investigation is described. In Section 3, a nonlinear observer of the Luenberger-type (NOL) for perspective noiseless linear systems is discussed and then after obtaining some important properties of such systems a convergence theorem of the nonlinear observer is presented. In Section 4, an Extended Kalman filter for general nonlinear systems is described. In Section 5, using computer simulation for a simple example, we examine the sensitivity to the noises of the nonlinear observer and the effectiveness of the Extended Kalman Filter (EKF). Finally, Section 6 gives some concluding remarks

2. PERSPECTIVE LINEAR SYSTEMS CORRUPTED BY NOISE

A perspective linear system corrupted by noise considered in this paper is described as

$$\begin{cases} \dot{x}(t) = Ax(t) + b(t) + Gw(t), \quad x(t_0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(Cx(t)) + v(t) \end{cases} \tag{1}$$

where $x(t) \in \boldsymbol{R}^n$ is the state, $b(t) \in \boldsymbol{R}^n$ the external input, $y(t) \in \boldsymbol{R}^n$ the *perspective observation* of the system, $A \in \boldsymbol{R}^{n\times n}$ a matrix and $C \in \boldsymbol{R}^{(m+1)\times n}$ a matrix with $m < n$ and $\mathrm{rank}\, C = m+1$. Further $h : \boldsymbol{R}^{m+1} \to \boldsymbol{R}^m$ is a nonlinear function of the form

$$\begin{cases} h(\xi) := \begin{bmatrix} \dfrac{\xi_1}{\xi_{m+1}} & \cdots & \dfrac{\xi_m}{\xi_{m+1}} \end{bmatrix}^T, \quad \xi_{m+1} \neq 0 \\ \xi = [\xi_1 \quad \cdots \quad \xi_m \quad \xi_{m+1}]^T \in \boldsymbol{R}^{m+1} \end{cases} \tag{2}$$

which produces the perspective observation, $w(t) \in \boldsymbol{R}^n$ and $v(t) \in \boldsymbol{R}^m$ are independent zero-mean Gaussian white noise processes. The problem of a simplified 3-dimensional noiseless vision considered in the previous works [3]-[10] can be described as a special case of System (1) with the perspective observation given as

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$$y = [y_1 \ y_2]^T = \begin{bmatrix} \dfrac{x_1}{x_3} & \dfrac{x_2}{x_3} \end{bmatrix}^T,$$

by setting $n = 3, m = 2$ and $C = I_3$ (the identity matrix) in the System (1).

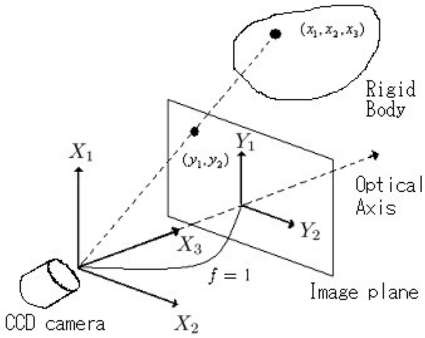


Fig 1. A Simple Perspective Observation

Next, we make various assumptions on System (1), which seem to be necessary and/or reasonable from the viewpoint of machine vision.

Assumption 1.

(i) $w(t)$, $v(t)$ are independent zero-mean Gaussian white noise such that

$$E\{w(t)\} = 0, \ E\{v(t)\} = 0$$

$$E\{\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^T(s) & v^T(s) \end{bmatrix}\} = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}\delta(t-s)$$

where $t \geq s$, $\delta(\cdot)$ is the Dirac delta function and

$$Q = Q^T \geq 0, \ R = R^T > 0.$$

(ii) x_0 is a random variable with known mean $\overline{x}_0$ and covariance P_0, i.e.,

$$E\{x_0\} = \overline{x}_0, \ E\{(x_0 - \overline{x}_0)(x_0 - \overline{x}_0)^T\} = P_0$$

(iii) x_0 is also independent of $w(t)$ and $v(t)$ in the sense

$$E\{w(t){x^T}_0\} = 0, \ \ E\{v(t){x_0}^T\} = 0, \ t \geq 0$$

Assumption 2. System (1) is assumed to satisfy the following conditions.

(i) System (1) is Lyapunov stable.

(ii) The observation vector $y(t)$ is a continuous and bounded function of t, that is,

$$y(\cdot) \in C^m[0, \infty) \cap L^m_\infty[0, \infty).$$

(iii) Express the set $\sigma(A)$ of all eigenvalues of A as $\sigma(A) = \sigma_-(A) \cup \sigma_0(A)$ where $\sigma_-(A)$ and $\sigma_0(A)$ $\sigma(A) = \sigma_-(A) \cup \sigma_0(A)$ where $\sigma_-(A)$ and $\sigma_0(A)$ indicate the sets of eigenvalues with strictly negative real part and zero real part, respectively. Let $W_-, \ W_0 \subset \boldsymbol{C}^n$ denote the generalized eigenspaces corresponding to $\sigma_-(A)$ and $\sigma_0(A)$, respectively, and choose a basis matrix $E_0 = [\xi_1 \ \cdots \ \xi_r]$ for W_0 where $r := \dim W_0$. Then, there exist $\overline{T} > 0$ and $\varepsilon > 0$ such that

$$\int_0^{\overline{T}} E_0^T e^{A^T\tau} C^T B^T(y(t+\tau)) \times B(y(t+\tau)) C e^{A\tau} E_0 d\tau \geq \varepsilon I_r, \ \forall t \geq 0. \quad (3)$$

□

Remark 3. All the conditions given in Assumption 2 are necessary and/or reasonable requirements from the viewpoint of machine vision.

(i) The condition (i) is imposed to ensure that if $b(t) \equiv 0$ then the motion of a moving body takes place within a bounded space.

(ii) The condition (ii) is imposed to ensure that the motion is smooth enough and takes place inside a conical region centered at the camera to produce a continuous and bounded measurement $y(t)$ on the image plane.

(iii) The condition (iii) is imposed to ensure some sort of detectability for the perspective system (1). In fact, the inequality (3) implies that (C, A) is a detectable pair and the external input $b(t)$ must not be identically zero. These facts are verified in Proposition 4. □

The following proposition gives some system theoretical implications of Assumption 2. (iii).

Proposition 4. Assume that System (1) is Lyapunov stable, let A_0 denote the critically stable part (i.e., the part corresponding to $\sigma_0(A)$) of A and set $C_0 := CE_0$. If Assumption 2. (iii) is satisfied, then the following statements hold true.

(i) (C, A) is a detectable pair, that is, the critically stable part (C_0, A_0) of (C, A) is observable.

(ii) The external input $b(t)$ is never identically zero. □

3. LUENBERGER-TYPE NONLINEAR OBSERVERS

In this section, we propose a Luenberger-type observer for a noiseless perspective linear system of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + b(t), & x(t_0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(Cx(t)) \end{cases} \quad (4)$$

First, notice that, denoting an estimate of the state $x(t)$ by $\hat{x}(t)$ a full-order state observer for System (4) generally has the form

$$\frac{d}{dt}\hat{x}(t) = \varphi(\hat{x}(t), b(t), y(t)), \quad \hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n \quad (5)$$

and satisfies the condition that whenever $\hat{x}(0) = x(0)$ the solution $\hat{x}(t)$ of (5) coincides completely with the solution $x(t)$ of System (4) for any $b(\cdot)$. Therefore, under the condition that (5) has a unique solution, it is possible to assume that the function $\varphi(\hat{x}, v, y)$ has the form

$$\varphi(\hat{x},b,y) = A\hat{x} + b + r(\hat{x},y)$$

where $r(\hat{x},y)$ is any sufficiently smooth function satisfying the condition $r(x,h(Cx)) = 0$ for all $x \in \boldsymbol{R}^n$. Among many functions $r(\hat{x},y)$ satisfying this condition, it would be wise to choose a function, which is reasonably simple, but has sufficient freedom to adjust its characteristics. As such a function, it is possible to choose

$$r(\hat{x},y) = K(y,\hat{x})[y - h(C\hat{x})],$$

where $K(y,\hat{x})$ is any sufficiently smooth function. Hence, it is possible to consider an observer of the Luenberger-type given as

$$\begin{cases} \dfrac{d}{dt}\hat{x}(t) = A\hat{x}(t) + b(t) + K(y(t),\hat{x}(t))[y(t) - h(C\hat{x}(t))] \\ \hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n \end{cases} \quad (6)$$

where $K(y,\hat{x})$ is a suitable matrix-valued function $K : \boldsymbol{R}^m \times \boldsymbol{R}^n \to \boldsymbol{R}^{n\times m}$, called an observer gain matrix.

The next important step is how to choose a gain matrix $K(y,\hat{x})$ in (6). To do so, first letting

$$\begin{cases} \xi = \begin{bmatrix} \xi_1 & \cdots & \xi_m & \xi_{m+1} \end{bmatrix}^T := Cx, \\ \hat{\xi} = \begin{bmatrix} \hat{\xi}_1 & \cdots & \hat{\xi}_m & \hat{\xi}_{m+1} \end{bmatrix}^T := C\hat{x}, \\ C = \begin{bmatrix} C_1^T & \cdots & C_m^T & C_{m+1}^T \end{bmatrix}^T \in \boldsymbol{R}^{(m+1)\times n} \end{cases}$$

and using output equation of System (4), one can easily obtain

$$\begin{aligned} y - h(C\hat{x}) &= h(Cx) - h(C\hat{x}) \\ &= \begin{bmatrix} \dfrac{\xi_1}{\xi_{m+1}} - \dfrac{\hat{\xi}_1}{\hat{\xi}_{m+1}} & \cdots & \dfrac{\xi_m}{\xi_{m+1}} - \dfrac{\hat{\xi}_m}{\hat{\xi}_{m+1}} \end{bmatrix}^T \\ &= \frac{1}{C_{m+1}\hat{x}} \begin{bmatrix} 1 & 0 & \cdots & 0 & -y_1 \\ 0 & 1 & \cdots & 0 & -y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -y_m \end{bmatrix} \begin{bmatrix} C_1(x-\hat{x}) \\ C_2(x-\hat{x}) \\ \vdots \\ C_{m+1}(x-\hat{x}) \end{bmatrix} \\ &= \frac{1}{C_{m+1}\hat{x}} B(y)Ce \end{aligned} \quad (7)$$

where $B(y)$ is the matrix-valued function given by

$$B(y) := \begin{bmatrix} I_m & -y \end{bmatrix} \in \boldsymbol{R}^{m\times(m+1)} \quad (8)$$

and $\rho \in \boldsymbol{R}^n$ is the estimation error vector defined by

$$e := x - \hat{x}. \quad (9)$$

Now, using (4) and (6) together with (9),

$$\frac{d}{dt}e(t) = \left\{ A - \frac{1}{C_{m+1}\hat{x}(t)} K(y(t),\hat{x}(t))B(y(t))C \right\} e(t)$$

where $e(t_0) = x(0) - \hat{x}(0) \in \boldsymbol{R}^n$. To eliminate the singularity in (6), let us choose a gain matrix $K(y,\hat{x})$ of the form

$$K(y,\hat{x}) := C_{m+1}\hat{x} \quad P^{-1}C^*B^*(y) \quad (10)$$

where $P \in \boldsymbol{R}^{n\times n}$ is an appropriately chosen matrix, which is considered to be a free parameter for the gain matrix.

Theorem.5 (Nonlinear Observers).

Assume that System (4) satisfies Assumption 2. and consider a nonlinear observer of the Luenberger-type (6), that is,

$$\begin{cases} \dfrac{d}{dt}\hat{x}(t) = A\hat{x}(t) + b(t) + K(y(t),\hat{x}(t))[y(t) - h(C\hat{x}(t))] \\ \hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n \end{cases} \quad (11)$$

where the gain matrix is given by (10), that is,

$$K(y,\hat{x}) := C_{m+1}\hat{x} \quad P^{-1}C^*B^*(y),$$

and $B(y)$ is given by (8).

Further, let $\pi_- : \boldsymbol{C}^n \to W_-$, $\pi_0 : \boldsymbol{C}^n \to W_0$ denote the matrix representations of the projection operators along W_0, W_-, respectively, and $P \in \boldsymbol{R}^{n\times n}$ be a symmetric positive definite matrix satisfying the Lyapunov inequality

$$A^*P + PA \leq -a\pi_-^*\pi_- \quad (12)$$

where $a > 0$ is a constant.

Then, the following statements hold.

(i) The estimation error $e(t) := x(t) - \hat{x}(t)$ satisfies the differential equation

$$\begin{cases} \dfrac{d}{dt}e(t) = [A - P^{-1}C^*B^*(y(t))B(y(t))C]e(t), \\ e(0) \in \boldsymbol{R}^n. \end{cases}$$

(ii) $e(t)$ converges exponentially to zero, that is, there exist $\alpha > 0, \beta > 0$ such that

$$\|e(t)\| := \|x(t) - \hat{x}(t)\| \leq \beta e^{-\alpha t}\|e(0)\|, \quad \forall t \geq 0.$$

□

4. EXTENDED KALMAN FILTER

In this section, we summarize the Extended Kalman Filter for are following systems

$$\begin{cases} \dot{x}(t) = f(x(t),u(t)) + G(t)w(t), \quad x(t_0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(x(t)) + v(t) \end{cases}$$

where $x(t) \in \boldsymbol{R}^n$ is the state, $u(t) \in \boldsymbol{R}^p$ is the input, $y(t) \in \boldsymbol{R}^n$ is the perspective observation of the system, $f(x,u) \in \boldsymbol{R}^n$, $h(x) \in \boldsymbol{R}^m$ are smooth nonlinear functions, and $G(t) \in \boldsymbol{R}^{n\times n}$ is matrix, $w(t) \in \boldsymbol{R}^n$ and $v(t) \in \boldsymbol{R}^m$ are is the system and observation noises, respectively satisfying the Assumption 1.

Theorem 5 (Extended Kalman Filter).

(i) Kalman filtering process

$$\dot{\hat{x}}(t) = f(\hat{x}(t),u(t)) + K_e(t)(y(t) - h(\hat{x}(t)))$$

(ii) Kalman gain matrix $K_e(t)$ is given by this form

$$K_e(t) := P_e(t)C_e^T(t)R^{-1}(t)$$

(iii) Matrix $P_e(t)$ is a solution of the following Riccati equation

$$\dot{P}_e(t) := A_e(t)P_e(t) + P_e(t)A_e^T(t) + G(t)Q(t)G^T(t) - K_e(t)C_e(t)P_e(t)$$

(iv) The matrices $A_e(t), C_e(t)$ are can be calculate from

$$A_e(t) = \frac{\partial f(x(t),u(t))}{\partial x(t)}\Big|_{x(t)=\hat{x}(t)}, \quad C_e(t) = \frac{\partial h(x(t))}{\partial x(t)}\Big|_{x(t)=\hat{x}(t)}. \tag{13}$$

(v) Initial conditions are given by

$$\hat{x}(t_0) := E\{x_0\} = \bar{x}_0(t),$$
$$P_e(t_0) := E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = P_{e0} \tag{14}$$

□

5. SIMULATION

In this section, using computer simulation for a simple example, we examine the sensitivity to the noises of the nonlinear observer and the effectiveness of the Extended Kalman Filter.

Example. We consider is a two degree-of-freedom system, consisting of two masses m_1, m_2 and two springs with the constants k_1, k_2 connected as shown in Fig. 2. The position vectors of masses m_1, m_2 are represented by $[\xi_1\ \eta_1]^T$, $[\xi_2\ \eta_2]^T$, respectively. Assume that the motion takes place only in the ξ-direction with $\eta_1 = \eta_2 \equiv \eta_c$, and only the motion of mass m_2 is observed via the CCD camera whose optical axis rotated by θ radian form η-axis as shown in the figure.

Now, assuming that an external force $u(t) = q\sin\omega t$ is applied to the mass m_1 and introducing the state variables

$$x_1 := \xi_1,\ x_2 := \dot{\xi}_1,\ x_3 := \xi_2,\ x_4 := \dot{\xi}_2,\ x_5 := \eta_1 = \eta_2 \equiv \eta_c,$$

the perspective linear system is easily described as

$$\begin{cases} \dot{x}(t) = Ax(t) + v(t), & x(0) = \bar{x} \in \boldsymbol{R}^5 \\ y(t) = h(Cx(t)) \end{cases}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & 0 & \frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad v(t) = \begin{bmatrix} 0 \\ \frac{k_1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t),$$

$$C = \begin{bmatrix} 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$y = h(Cx) = \frac{x_3\cos\theta - x_5\sin\theta}{x_3\sin\theta + x_5\cos\theta}.$$

Then it is not difficult to check that all the conditions in Assumption 2. except the inequality (3) are satisfied, and that A is Lyapunov stable with $\sigma_-(A) = \phi$ and hence $\pi_-^*\pi_- = 0$ so that the Lyapunov inequality (10) becomes

$$A^*P + PA \le -a\pi_-^*\pi_- = 0.$$

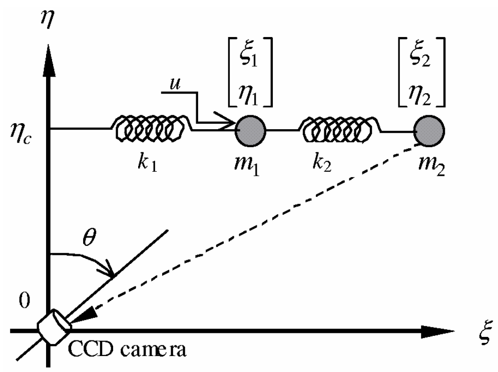


Fig. 2. A two-degree-of-freedom system

Next, the numerical values for simulation are set as follows: For the perspective system,

$$m_1 = 2,\ m_2 = 1,\ k_1 = 2,\ k_2 = 1,\ \theta = 0.5\ \text{[rad]},\ \omega = 1\ \text{[rad]},$$
$$q = 1,\ x(0) = [0.1\quad 0.2\quad 0.3\quad 0.1\quad 0.5]^T$$

and for the observer,

$$P^{-1} = \begin{bmatrix} 1.1665 & -0.0000 & -0.6905 & 0.0000 & 0 \\ -0.0000 & 2.0950 & -0.0000 & -1.8571 & 0 \\ -0.6905 & -0.0000 & 1.6425 & -0.0000 & 0 \\ 0.0000 & -1.8571 & -0.0000 & 2.3330 & 0 \\ 0 & 0 & 0 & 0 & 0.2562 \end{bmatrix}$$

where P is a suitably chosen solution of the matrix equation

$$A^*P + PA = 0.$$

Next, we set or calculate some initial data for Extended Kalman Filter in Theorem 5. Linearization system matrices $A_e(t), C_e(t)$ are calculated by (13), i.e.,

$$A_e(t) = A,$$
$$C_e(t) = \frac{1}{(\hat{x}_3(t)\sin\theta + \hat{x}_5(t)\cos\theta)^2}\begin{bmatrix} 0 & 0 & \hat{x}_3(t) & 0 & -\hat{x}_5(t) \end{bmatrix}.$$

the matrix P_{e0} is can get from (14), we will do four time simulations for a deferent case noise intensities in Tab. 1 and deferent initial states,

$$\hat{x}_{10} = \begin{bmatrix} 0.3 & 0.0 & 0.5 & 0.0 & 0.9 \end{bmatrix}^T,$$
$$\|e_{10}\| := \|x_0 - \hat{x}_{10}\| = 0.5385$$

$$\hat{x}_{20} = \begin{bmatrix} 0.5 & 0.0 & 1.0 & 0.0 & 1.0 \end{bmatrix}^T$$
$$\|e_{20}\| := \|x_0 - \hat{x}_{20}\| = 0.9747$$

	I	II	III	IV
Q	$10^{-6}I$	$10^{-6}I$	$10^{-4}I$	$10^{-4}I$
R	10^{-2}	10^{-2}	10^{-2}	10^{-2}
$\|e(t_0)\|$	0.5385	0.9747	0.5385	0.9747

Table 1. Data for extended Kalman filter

The time evolutions of each component of the estimation error $e(t) = x(t) - \hat{x}(t)$ for simulations I, II, III, IV are depicted in Fig. 3-6, and the results summarized on the Tab. 2 show that the observer works well than the Extended Kalman Filter in this simulation.

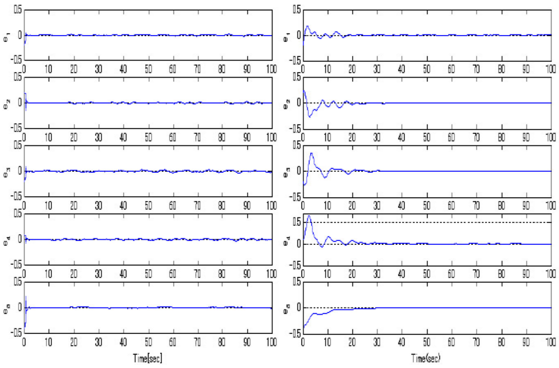
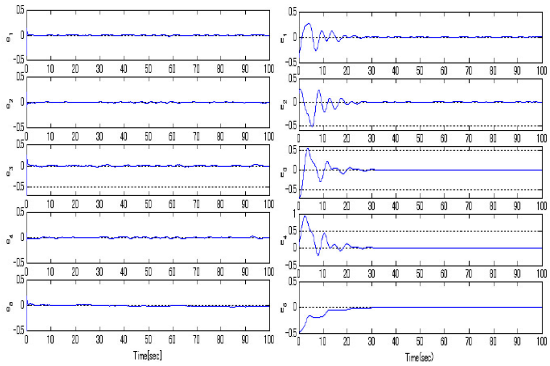
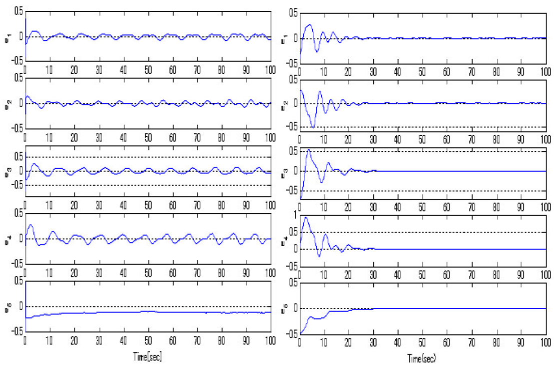


Fig. 3. Each component of the estimation error $e(t)$ for the case I. (the left one is Extended Kalman Filter, and the right one is Nonlinear Observer).

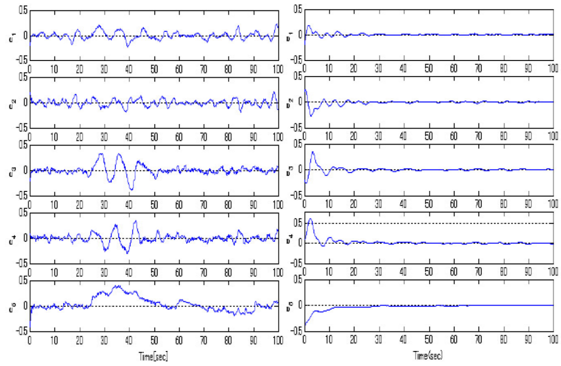


Convergent case (for the same settings)

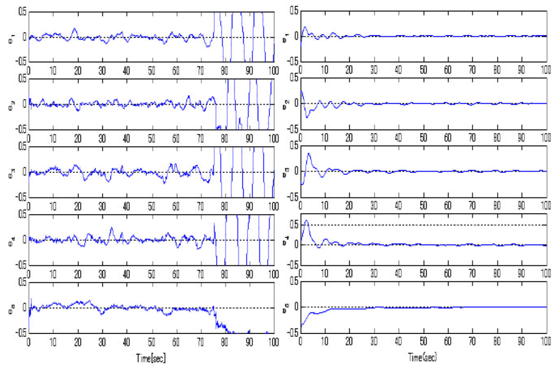


Non convergence case(for the same settings)

Fig. 4 Each component of the estimation error $e(t)$ for the case II. (left one is Extended Kalman Filter, right one is Nonlinear Observer).

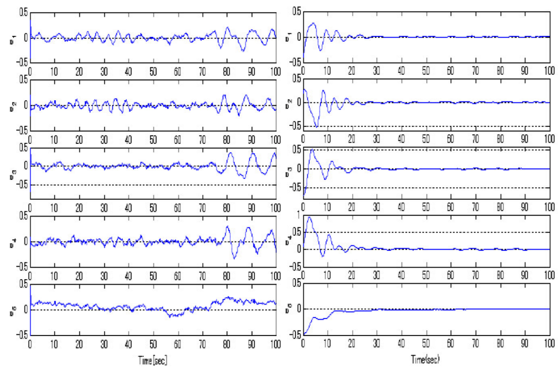


Convergence case(for the same settings)

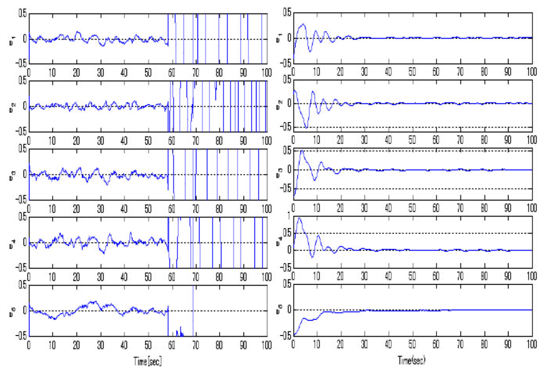


Non convergence case(for the same settings)

Fig. 5 Each component of the estimation error $e(t)$ for the case III. (left one is Extended Kalman Filter, right one is Nonlinear Observer)



Convergence case (for the same settings)



Non convergence case (for the same settings)

Fig. 6 Each component of the estimation error $e(t)$ for the case IV. (left one is Extended Kalman Filter, right one is Nonlinear Observer)

	I	II	III	IV	convergence
EKF	○	△	△	△	Fast
NOL	○	○	○	○	slow

Table 2. The results of the simulations.

6. CONCLUDINGS

This paper studied the state estimation problem for a rigid body moving in three dimensional spaces using the image data obtained by perspective observations. By means of computer simulations for a simple example Fig 2, we examine the sensitivity to the noises of the nonlinear observer developed in the recent paper linear time-invariant systems arising in machine vision [1] and the effectiveness of the Extended Kalman Filter. As a result, it is possible to summarize from Table 2. as follows

- Nonlinear observers considerably have robustness for both the system noise and the observation noise.
- Robustness to the noise of the Kalman filter is considerably small compared with a nonlinear observer. Especially, robustness to the system noise is very small.

REFERENCES

[1] R. Abdursul, H. Inaba and B. K. Ghosh, Luenberger-type observers for perspective linear systems, Proceedings of the European Control Conference, CD-ROM, Porto, Portugal, September 2001.

[2] K. Reif, S. Gunther, E. Yaz and R. Unbehauen, Stochastic stability of the continuous-time extended Kalman filter, IEE proc, Control Theory Appl., Vol.147, No.1, 2000.

[3] B. K. Ghosh and E. P. Loucks, A perspective theory for motion and shape estimation in machine vision, SIAM Journal on Control and Optimization, vol.35, pp.1530-1559, 1995.

[4] B. K. Ghosh, M. Jankovic and Y. T. Wu, Perspective problems in system theory and its applications to machine vision, Journal of Mathematical Systems, Estimation and Control, vol.4, pp.3-38, 1994.

[5] B. K. Ghosh, H. Inaba and S. Takahashi, Identification of Riccati dynamics under perspective and orthographic observations, IEEE Transactions on Automatic Control, vol. 45, pp. 1267-1278, 2000.

[6] A. Chiuso, P. Favaro, H. Jin and S. Soatto, Structure from motion causally integrated over time, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 24, no. 4, pp. 523-535, 2002.

[7] W. P. Dayawansa, B. K. Ghosh, C. Martin and X. Wang, A necessary and sufficient condition for the perspective observability problem, Systems and Control Letters, vol.25, pp.159-166, 1993.

[8] H. Inaba, A. Yoshida, R. Abdursul and B. K. Ghosh, Observability of perspective dynamical systems, IEEE Proceedings of Conference on Decision and Control, pp.5157-5162, Sydney, Australia, December 2000.

[9] A. Matveev, X. Hu, R. Frezza and H. Rehbinder, Observers for systems with implicit output, IEEE Transactions on Automatic Control, vol. 45, pp.168-173, 2000.

[10] S. Soatto, R. Frezza and P. Perona, Motion estimation via dynamic vision, IEEE Transactions on Automatic Control, vol. 41, pp.393-413, 1996.

[11] M. Jankovic and B. K. Ghosh, Visually guided ranging from observations of points, lines and curves via an identifier based nonlinear observer, Systems and Control Letters, vol. 25, pp. 63-73, 1995.

[12] B. K. Ghosh and J. Rosenthal, A generalized Popov-Belevitch-Hautus test of observability, IEEE Transactions on Automatic Control, vol. 40, pp.176-1180, 1995.