Output feedback model predictive control for Wiener model with parameter dependent Lyapunov function

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Abstract: In this paper, we consider a robust output feedback model predictive controller(MPC) design for Wiener model. Nonlinearities that couldn't be represented in static nonlinearity block of Wiener model are regarded as uncertainties in linear block. An dynamic output feedback controller design method is presented for Wiener MPC. According to MPC algorithm, the control law is computed based on linear matrix inequality(LMI)at each sampling time by solving convex optimization. Also, a new parameter dependent Lyapunov function is proposed to get a less conservative condition. The results are illustrated with numerical example.

Keywords: Model predictive control, Wiener model, Dynamic output feedback, Parameter dependent Lyapunov function

1. Introduction

There are very few design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints. Wiener model is a representative model to treat the nonlinearities of a process without complications associated with general nonlinear operators. Wiener model consists of a dynamic linear block followed in series by a static nonlinear element and it adequately represent many of the nonlinearities commonly encountered in industrial processes[3]. There are several techniques to remove nonlinearity from the control problem. Norquay developed a relaxed method by inverting the static nonlinearity so that Wiener model could have been incorporated into model predictive control(MPC) scheme[3]. Bloemen presented another Wiener MPC by transforming the static nonlinearities into polytopic uncertainty descriptions[7]. These Wiener MPC algorithms are considered based on state feedback or state observer based output feedback. Moreover, there can be inseparable nonlinearities in the linear block i.e., we couldn't always express the all nonlinearities in a block following the linear block in series.

Since model predictive control can easily handle time varying system with input and output constraints, it is a popular technique for the slow dynamical system[1]. In standard MPC algorithm, the optimal control input is computed at each time instance by solving optimization problem over a fixed time horizon[1]. MPC uses estimated state based on approximation of model, which is not a real model, to calculate the optimal cost. Thus it is important for MPC to be robust to model uncertainty. Robust constrained MPC based on state feedback has been studied[1]. Also output feedback based robust constrained MPC has been developed but the control law is state observer based form[2]. A dynamic output feedback controller has been introduced for MPC using a common Lyapunov function based on linear matrix inequality(LMI) without concerning uncertainties[4]. In this paper, we introduce a dynamic output feedback controller for Wiener model and present robust constrained MPC using parameter dependent Lyapunove function.

This paper organized as follows. In section 2, we present Wiener model whose linear block is described in structured uncertain model and formulate the constrained MPC. In section 3, we derive the LMI conditions using parameter dependent Lyapunov function to solve the optimization problem. Also the dynamic output feedback controller is implemented using the variables from the result of LMI optimization. In section 4, a numerical example is presented to illustrate the design scheme. Finally, in section 5, we conclude this paper.

2. Problem statement

2.1. The development of Wiener model

A Wiener system illustrated in Fig 1 consists of the series connection of a dynamic linear system and a static nonlinearity.

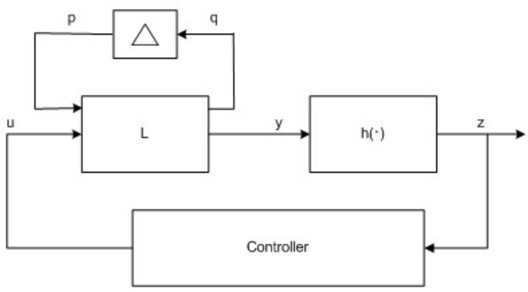

Fig. 1. Block Diagram of Wiener model

The linear block contains uncertainty in feedback loop and it can be represented by:

$$
\begin{aligned}
x(k+1) &= Ax(k) + B_1 p(k) + B_2 u(k) \\
q(k) &= C_1 x(k) + D_{11} p(k) \\
y(k) &= C_2 x(k) + D_{21} p(k) + D_{22} u(k) \\
p(k) &= \Delta(\theta(k)) q(k) \\
z(k) &= h(y(k))
\end{aligned}
\tag{1}
$$

where x(k)$\in R^n$ is the state of the plant, u(k)$\in R^m$ is the control input and y(k)$\in R^p$ is the output of the linear block

respectively. $h(\cdot)$ is the nonlinear mapping from y(k) to z(k) and $z(k)$ is the output of the nonlinear block.
The nonlinearity of the Wiener model is transformed into a polytopic uncertainty description. Without loss of generality assume that $h_1 \dots h_p$ are polynomials. The nonlinearity can be written as:

$$z(k) = h(y(k)) = H(y)y(k)$$

where

$$\begin{aligned} H(y) \in \Omega &= Co\{H_1, \dots, H_{2^p}\} \\ &= \Sigma_{i=1}^{2^p} \alpha_i H_i \\ \Sigma_i \alpha_i &= 1 \end{aligned} \tag{2}$$

in which $H(y)$ is a diagonal matrix because of the special structure of the nonlinearity. When the operating region for $y(k)$ is limited the entries of $H(y)$ are bounded by minimum and maximum values. All the possible combinations of the maximum and minimum values of the element of $H(y)$ are used to generate 2^p vertices $Co\{H_1, \dots, H_{2^p}\}$ of the polytopic description Ω which contains the nonlinear matrix $H(y)$.
(Co refers to the convex hull).

The operator Δ is block-diagonal:

$$\Delta = \begin{bmatrix} \Delta_1 & & & \\ & \Delta_2 & & \\ & & \ddots & \\ & & & \Delta_r \end{bmatrix} \tag{3}$$

with $\Delta_i : R^{n_i} \rightarrow R^{n_i}$. Δ can represent either a memoryless time-varying matrix with $\| \Delta_i(k) \|_2 \equiv \bar{\sigma}(\Delta_i(k)) \leq 1$, i=1,2,$\cdots$,r, k$\geq$0 or a convolution operator (e.g. a stable LTI dynamical system), with the operator norm induced by the truncated L2-norm less than 1, i.e.,

$$\begin{aligned} \textstyle\sum_{j=0}^{k} p_i(j)^T p_i(j) &\leq \sum_{j=0}^{k} q_i(j)^T q_i(j) \\ i = 1, \cdots, r_i, &\qquad \forall k \geq 0 \end{aligned} \tag{4}$$

2.2. Dynamic controller design

We want to find a control law at each time, k, for the system (1) with following representation:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c z(k) \\ u(k) &= C_c x_c(k) \end{aligned} \tag{5}$$

where $x_c(k+1) \in R^n$ and A_c, B_c, C_c are matrices of appropriate dimensions. The problem is redefined to the determination of the matrices A_c, B_c, C_c so that the closed-loop system is stable. The augmented system is represented by:

$$\begin{aligned} \bar{x}(k+1) &= \bar{A}\bar{x}(k) + \bar{B}p(k) \\ u(k) &= K\bar{x}(k) \\ q(k) &= \bar{C(x)}(k) + \bar{D}p(k) \end{aligned} \tag{6}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & B_2 C_c \\ B_c \sum \alpha_i H_i C_2 & A_c \end{bmatrix} \in R^{2n \times 2n} \\ \bar{B} &= \begin{bmatrix} B_1 \\ B_c \sum \alpha_i H_i D_{21} \end{bmatrix} \in R^{2n \times m} \\ \bar{C} &= \begin{bmatrix} C_1 & 0 \end{bmatrix} \in R^{p \times 2n} \\ \bar{D} &= D_{11} \\ K &= \begin{bmatrix} 0 & C_c \end{bmatrix} \end{aligned} \tag{7}$$

and

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix} \in R^{2n} \tag{8}$$

To guarantee stability, we consider an infinite horizon MPC problem[1]. The control law, at each step k, can be computed by minimizing the objective function given by:

$$\begin{aligned} J_\infty(k) = &\textstyle\sum_{i=0}^{\infty} [\bar{x}(k+i|k)^T \bar{Q} \bar{x}(k+i|k) \\ &+ u(k+i|k)^T R u(k+i|k)] \end{aligned} \tag{9}$$

where

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

$Q \geq 0, R > 0$, and $\bar{x}(k+i|k)$ denotes the augmented state predicted based on the measurements at time k, $\bar{x}(k|k) = \bar{x}(k)$.

Let us consider a quadratic function:

$$V(\bar{x}(k|k)) = \bar{x}(k|k)^T P(\theta(k)) \bar{x}(k|k), \quad P(\theta(k)) > 0 \tag{11}$$

where

$$P(\theta(k)) = \begin{bmatrix} I \\ \Delta_a(\theta(k)) \end{bmatrix}^T P_a(k) \begin{bmatrix} I \\ \Delta_a(\theta(k)) \end{bmatrix} \tag{12}$$

$$\Delta_a(\theta(k)) = (I - \Delta(\theta(k))D)^{-1} \Delta(\theta(k)) C \tag{13}$$

for all $\Delta(\theta(k)) \in \Delta$. It is the parameter dependent representation and we suppose the parameter can be measured.

For the cost monotonicity, let us suppose V satisfies the following inequality for all $x(k+i|k)$, $u(k+i|k)$, $i \geq 0$:

$$\begin{aligned} &V(k+j+1|k) - V(k+j|k) < \\ &-[x(k+j|k)^T \bar{Q} + u(k+j|k)^T R u(k+j|k)] \end{aligned} \tag{14}$$

By summing (14) from $i = 0$ to $i = \infty$, we obtain:

$$-V(x(k|k)) \leq -J_\infty(k) \tag{15}$$

that means the quadratic function, V, can be an upper bound for the objective function (9). Thus our goal is redefined to the minimization of (11).

3. Main result

Theorem 1: Consider the system (1) at time instant k. The dynamic output feedback control law (2) that minimize $J_\infty(k)$ can be solved by the following semi-definite programming :

$$min\ \ Tr(P_a) \tag{16}$$

subject to

$$\begin{pmatrix}
-X & * & * & * & * & * & * & * & * & * \\
-I & -Y & * & * & * & * & * & * & * & * \\
0 & 0 & -Z^{-1} & * & * & * & * & * & * & * \\
0 & 0 & 0 & -\Lambda & * & * & * & * & * & * \\
G & YA+F\Sigma\alpha_iH_iC_2 & YB_1+F\Sigma\alpha_iH_iD_{21} & 0 & -Y & * & * & * & * & * \\
AX+B_2L & A & 0 & 0 & -I & -X & * & * & * & * \\
0 & 0 & 0 & Z^{-1}\Lambda & 0 & 0 & -Z^{-1} & * & * & * \\
C_1AX+C_1B_2L & C_1A & C_1B_1 & D_{11}\Lambda & 0 & 0 & 0 & -\Lambda & * & * \\
Q^{1/2}X & Q^{1/2} & 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\
R^{1/2}L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{pmatrix} < 0 \tag{17}$$

Proof:
By substituting (6) and (12) into (14), we could obtain:

$$\begin{bmatrix} \bar{x} \\ p \\ w \end{bmatrix}^T \left\{ \begin{bmatrix} \bar{A}^T & 0 \\ \bar{B}^t & 0 \\ 0 & I \end{bmatrix} P_a \begin{bmatrix} \bar{A} & \bar{B} & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} P_a \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \right\} \begin{bmatrix} \bar{x} \\ p \\ w \end{bmatrix}$$
$$< - \begin{bmatrix} \bar{x} \\ p \\ w \end{bmatrix} \begin{bmatrix} \bar{Q}+K^TRK & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ p \\ w \end{bmatrix} \tag{18}$$

where

$$\begin{aligned}
\bar{x} &= \bar{x}(k+j|k), \quad p = \Delta_a(k)\bar{x}(k+j|k), \\
w &= \Delta_a(k+1)\bar{x}(k+j+1|k) \\
K &= [0 \quad C_c] \quad \bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{19}$$

This inequality holds for all nonzero vector $\begin{bmatrix} \bar{x}\ ; & p\ ; & w\ ; \end{bmatrix}$ satisfying

$$\begin{aligned}
w^Tw &= (\bar{A}\bar{x}+\bar{B}p)^T\Delta_a^T(k+1)\Delta_a(k+1)(\bar{A}\bar{x}+\bar{B}p) \\
&= (\bar{C}\bar{A}\bar{x}+\bar{C}\bar{B}p+\bar{D}w)^T\Delta^T\Delta(\bar{C}\bar{A}\bar{x}+\bar{C}\bar{B}p+\bar{D}w) \\
&\le (\bar{C}\bar{A}\bar{x}+\bar{C}\bar{B}p+\bar{D}w)^T(\bar{C}\bar{A}\bar{x}+\bar{C}\bar{B}p+\bar{D}w)
\end{aligned} \tag{20}$$

Remark 1: w denotes $p(k+j+1|k)$. By considering $p(k+j|k)$ and $p(k+j+1|k)$ together, we can take the rate bound of the uncertainty into account and it derives the less conservative condition.

Using S-procedure, it is easy to see that inequality (18) and constraint (20) are satisfied if

$$\begin{bmatrix} \underline{A}^TP_a\underline{A}-P_a+W & \underline{A}^TP_a\underline{B} \\ \underline{B}^TP_a\underline{A} & \underline{B}Pa\underline{B}-\Lambda^{-1} \end{bmatrix} + \begin{bmatrix} \underline{C}^T \\ \underline{D}^T \end{bmatrix} \Lambda^{-1} \begin{bmatrix} \underline{C} & \underline{D} \end{bmatrix} < 0 \tag{21}$$

where

$$\begin{gathered}
\underline{A} = \begin{bmatrix} \bar{A} & \bar{B} \\ 0 & 0 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} 0 \\ I \end{bmatrix} \\
\underline{C} = \begin{pmatrix} CA & CB \end{pmatrix} \quad \underline{D} = \bar{D} \\
W = \begin{bmatrix} \bar{Q}+K^TRK & 0 \\ 0 & 0 \end{bmatrix} \\
\Lambda = \tau I
\end{gathered} \tag{22}$$

By Congruence transformation with $diag\{I,\Lambda,I,I\}$ after Schur's complement, (26) is equivalent to:

$$\begin{bmatrix} -P_a+W & * & * & * \\ 0 & -\Lambda^{-1} & * & * \\ \underline{A} & \underline{B}\Lambda & -Q_a & * \\ \underline{C} & \underline{D}\Lambda & 0 & -\Lambda \end{bmatrix} < 0 \tag{23}$$

where

$$Q_a \quad = \quad P_a^{-1} \tag{24}$$

Let us partition matrices Q_a and P_a in the form:

$$Q_a = \begin{bmatrix} X & U & 0 \\ U^T & \hat{X} & 0 \\ 0 & 0 & Z \end{bmatrix} \quad P_a = \begin{bmatrix} Y & V & 0 \\ V^T & \hat{Y} & 0 \\ 0 & 0 & Z^{-1} \end{bmatrix} \tag{25}$$

and define the matrices:

$$T_1 = \begin{bmatrix} X & I & 0 \\ U^T & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \quad T_2 = \begin{bmatrix} Y & I & 0 \\ V^T & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \tag{26}$$

By pre-multiplying by T^T and post-multiplying by T, with :

$$T = \begin{bmatrix} T_1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{27}$$

we obtain:

$$\begin{bmatrix} -T_1^T P_a T + T_1^T W T & * & * & * \\ 0 & -\Lambda & * & * \\ T_2^T \underline{A} T_1 & T_2^T \underline{B}\Lambda & -T_2^T Q_a T_2 & * \\ \underline{C} T_1 & \underline{D}\Lambda & 0 & -\Lambda \end{bmatrix} < 0 \quad (28)$$

By Substituting (7),(22),(25) and (26) into (28), we obtain (17) where:

$$\begin{aligned} F &= VB_c \\ L &= C_c U^T \\ G &= YAX + F\sum \alpha_i H_i C_2 X + YB_2 L + VA_C U^T \end{aligned} \quad (29)$$

By solving (17), we obtain the value of variables X,Y,Z,F,L and G. Therefore the controller is given by:

$$\begin{aligned} V &= (I - YX)(U^T)^{-1} \\ B_c &= V^{-1}F \\ C_c &= L(U^T)^{-1} \\ A_c &= V^{-1}\bar{G}(U^T)^{-1} \end{aligned} \quad (30)$$

where $\bar{G} = G - YAX - F\sum \alpha_i H_i C_2 X - YB_2 L$.

4. Numerical example

Consider angular positioning system described in Fig2.

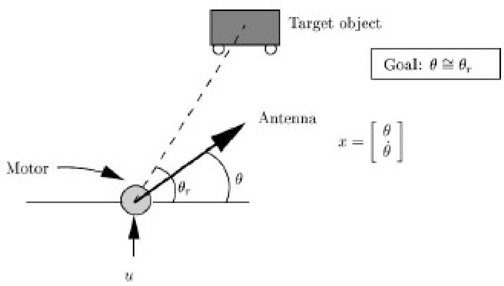

Fig. 2. Angular position system

It consists of the linear block described by:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} \theta(k+1) \\ \dot{\theta}(k+1) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0.1 \\ 0 & 1 - 0.1\alpha(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.08 \end{bmatrix} u(k) \\ &\equiv A(k)x(k) + B(k)u(k) \end{aligned} \quad (31)$$

and nonlinearity in the output, that is polytope with known bounds:

$$0.5 \leq H \leq 1.5 \quad (32)$$

$\alpha(k)$ is a nonlinearity expressed by sine function, which cannot be separated from linear block. Regarding it as an uncertainty, the system can be interpreted as a structured uncertain model by :

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0.1 \\ 0 & 0.495 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0.08 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 0 & 0.495 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ D_{11} &= 0 \qquad D_{21} = 0 \\ Q &= \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \quad R = 0.1 \end{aligned} \quad (33)$$

with the initial condition $x(0) = [0.05 \quad 0]^T$, $x_c(0) = [0 \quad 0]^T$ Fig.3 shows the output and the control trajectory of the constrained MPC. The dashed line is the result of a MPC using a common Lyapunov function presented in [4] and the solid line shows the case of using the parameter dependent Lyapunov function presented above. Compared to the case of common Lyapunov function, the improved result can be obtain by the parameter dependent Lyapunov function.

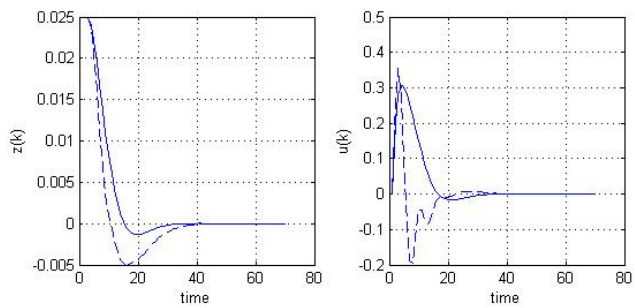

Fig. 3. Closed-loop response for the Wiener model: solid lines shows the case of parameter dependent Lyapunov function ; dashed lines shows the case of a common Lyapunov function

5. Conclusion

We have constructed a dynamic output feedback controller for Wiener model composed of dynamic linear block ,static nonlinear block and another nonlinearity which couldn't have been apart from linear block. That nonlinearity has been considered as an uncertainty in the linear block. The MPC algorithm have been adopted to control the entire Wiener model. Also, we have proposed the parameter dependent Lyapunov function. LMI technique has been used to solve the optimization problem. A numerical example has been represented performance of the parameter dependent Lyapunov function and the result has shown rather improved closed-loop response than the case of a common Lyapunove function.

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