Adaptive Double Notch Filter for Interference Suppression in the GPS Receiver

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Abstract: In this paper, an efficient scheme of the adaptive notch filter is presented for rejecting the narrow bandwidth interferences(NBI) in GPS receiver. Designed is the lattice IIR double notch filter for more efficient suppression of the NBI with less computational complexity. The algorithm is of recursive prediction error form and uses a special constrained model of IIR with a minimal number of parameters. This paper chooses seven different jamming scenarios including one without jamming for evaluating the proposed filter algorithm. The simulation results to the jamming scenarios show that the proposed algorithm adjusts the double notch filter effectively for the given JSR, and provides better SNR than the conventional algorithms. Finally, it is shown that the advantages of the proposed filter algorithm can range as high as JSR 79dB in time domain processing. Also, the ADNF(adaptive double notch filter) guarantees that more than SNR 10dB of GPS receiver can be always maintained. In conclusion, there is enough evidence to believe that the proposed algorithm will perform quite well for removing interference signals.

Keywords: GPS, Interference Suppression, Estimation, Anti-Jamming, EW, ECCM

1. INTRODUCTION

The GPS navigation satellite system is designed to serve both military and commercial needs. Because of its EW(Electronic Warfare) applications, the ability to tolerate significant amounts to interference and jamming was an important consideration in the design of the signal structure. Obviously, any radio-navigation system can be disrupted by an interference of sufficiently high power, and the GPS is no exception. So the DSSS(direct sequence spread spectrum) technique in the GPS employs the PRN(pseudo random noise) to spread the spectrum of data sequence over a much wider bandwidth than required. But when the interference is too strong to be suppressed by the given processing gain of the spread spectrum system, the GPS receiver no longer operates properly. More specifically, it has been shown that NBI(Narrow Bandwidth Interference) suppression capability of the GPS receiver can be further enhanced by employing adaptive filters prior to dispreading[1, 2].

In this paper, an efficient filter algorithm is presented for adaptive notch filtering in GPS receiver. Precisely this paper employs the lattice IIR double notch filter for more efficient suppression of the narrow bandwidth interference with less computational complexity. The algorithm is of recursive prediction error(RPE) form and uses a special constrained model of infinite impulse response(IIR) with a minimal number of parameters.

This paper is organized as follows. Section 2 designs the optimal IIR notch filter. In Section 3, the lattice frequency estimation algorithm is derived. In Section 4, the proposed interference suppression system is presented. The adaptive double notch filter is also illustrated in detail. Simulation results are provided in Section 5 and finally conclusions are given in Section 6.

2. OPTIMAL LATTICE NOTCH FILTER DESIGN

In this section, the optimal lattice IIR notch filter is designed for using ADNF. The transfer function of the second order direct form IIR notch filter can be expressed as Eq.(1). The IIR filter with getting same position pole-zero is designed for that the optimal notch filter removes frequency factor ω_0 in some signals. As the narrowband signals approach sinusoidal signals, the zeros and poles approach the unit circle. In the limit, the poles and zeros are on the unit circle at the same locations. However, for stability, the poles have to be kept away from the unit circle[3, 4].

$$H(z) = \frac{(1-e^{jw_0}z^{-1})(1-e^{-jw_0}z^{-1})}{(1-\alpha\cdot e^{jw_0}z^{-1})(1-\alpha\cdot e^{-jw_0}z^{-1})} = \frac{1+2\cdot a_0\cdot z^{-1}+z^{-2}}{1+2\cdot\alpha\cdot a_0\cdot z^{-1}+\alpha^2\cdot z^{-2}} \quad (1)$$

In Eq.(1), α is the pole-zero contraction factor that adjusts the width of the notch, and $a_0 = -\cos(w_0)$ determines the notch frequency. The value of α closer to 1 implies the narrower notch. The frequency response and pole-zero plot is described as Fig.1 by α . But it is shown that the lattice form notch filter provides better convergence properties, and more accurate frequency estimate compared to the direct form implementations[6].

$$H(z) = \frac{1+2\cdot a_0\cdot z^{-1}+z^{-2}}{1+a_0\cdot(1+\alpha)\cdot z^{-1}+\alpha\cdot z^{-2}} \quad (2)$$

In Eq.(2), it is shown that the IIR notch filter of lattice type yields the unbiased frequency estimate regardless of the pole-zero contraction factor and the noise variance, whereas the direct form of Eq.(1) yields the biased estimate. However, both Eq.(1) and Eq.(2) are more or less direct implementations of the rational fractional transfer function. With increasing filter order the direct realization of the transfer function becomes more and more critical with respect to noise performance, stability and coefficient sensitivity. To avoid these problems, the transition to state-space structures which require a higher implementation complexity but offer more flexibility with respect to the control of the named drawbacks. In this paper, because the ladder filters exhibit excellent properties with respect to coefficient sensitivity, which renders them very appropriate to act as a model for estimation implementations. As stated previously, there can be many implementation schemes of realizing the transfer function in Eq.(2). But we implement the transfer function by cascading all-pole and all-zero lattice filters as shown in Fig.(2)[3].

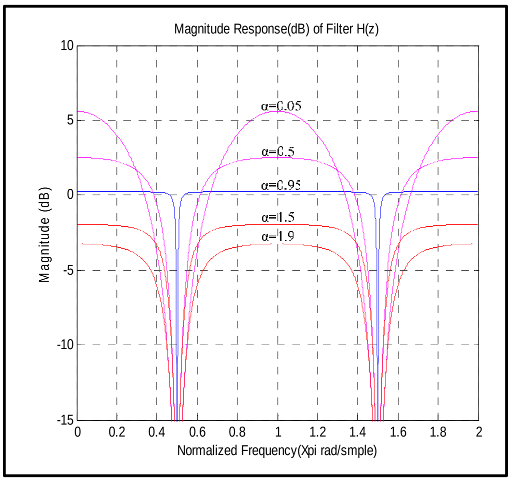

Fig. 1 The frequency response of optimal IIR notch filter

Through transform method of lattice structure of discovered by Gray and Markel, the lattice structure is showed using two-multiplier lattice and reversion of the two-multiplier lattice as Fig.(2). Eq.(3) is derived from the optimal IIR notch filter of lattice structure[7].

$$H(z)=\frac{1+2\cdot k_0\cdot z^{-1}+k_1\cdot z^{-2}}{1+k_0\cdot(1+a_1)\cdot z^{-1}+a_1\cdot z^{-2}} \qquad (3)$$

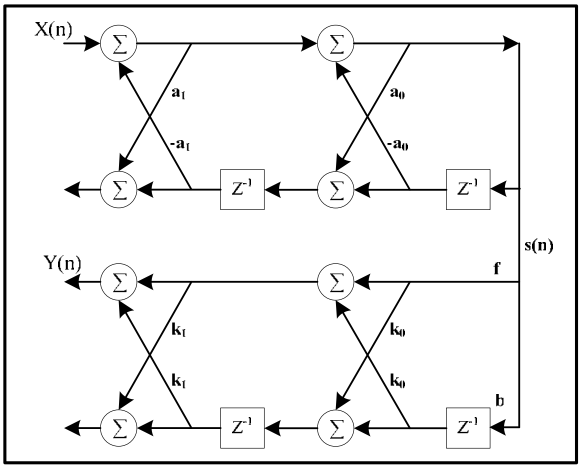

Fig. 2 The structure of the lattice IIR notch filter

3. ADAPTIVE LATTICE ESTIMATOR DESIGN

For the derivation of the adaptive algorithm in this section, consider the frequency estimator consists of m stages connected in cascade as in Fig.4. For presentation of ALE in Fig.2, these relations are reproduced here in the expanded form.

$$\begin{aligned} f_m(n) &= f_{m-1}(n)+k_m^* b_{m-1}(n-1) \\ b_m(n) &= b_{m-1}(n-1)+k_m f_{m-1}(n) \end{aligned} \qquad (4)$$

In Eq.(4), $m=1, 2, 3, \ldots, M$ and M is the final order of the predictor. Specifically, the reflection coefficient k_m is chosen so as to minimize the sum of the mean-squared values of the forward and backward prediction errors. Let the cost function J_m denote this sum at the output of stage m of the lattice predictor.

$$J_m = E[|f_m(n)|^2]+E[|b_m(n)|^2] \qquad (5)$$

Substituting Eq.(4) in Eq.(5), we get the cost function J_m as Eq.(6).

$$\begin{aligned} J_m &= \{E[|f_{m-1}(n)|^2]+E[|b_{m-1}(n-1)|^2]\}\cdot[1+|k_m|^2] \\ &+2k_m E[f_{m-1}(n)b_{m-1}^*(n-1)]+2k_m^* E[b_{m-1}(n-1)f_{m-1}^*(n)] \end{aligned} \qquad (6)$$

In general, the reflection coefficient k_m is complex valued, as shown by

$$k_m=\alpha_m+j\beta_m \qquad (7)$$

Therefore, differentiating the cost function J_m with respect to both the real and imaginary parts of k_m, we get the complex-valued gradient.

$$\nabla J_m=\frac{\partial J_m}{\partial\alpha_m}+j\frac{\partial J_m}{\partial\beta_m}=0 \qquad (8)$$

Putting this gradient to zero, we find that the optimum value of the reflection coefficient, for which the cost function J_m is minimum, equals Eq.(9)

$$k_{m,o}=-\frac{2E[b_{m-1}(n-1)f_{m-1}^*(n)]}{E[|f_{m-1}(n)|^2+|b_{m-1}(n-1)|^2]} \quad (m=1,2,\ldots,M) \qquad (9)$$

It derives Eq.(9) from Eq.(8) for the reflection coefficient. Its use offer two interesting properties. The reflection coefficient $k_{m,o}$ satisfies the condition of Eq.(10) as a minimum-phase design for the ALE.

$$|k_{m,o}|\le 1 \qquad for\ all\ m \qquad (10)$$

The mean-square values of the forward and backward prediction errors at the output of stage m are related to those at its own input as follows.

$$E[|f_m(n)|^2] = \left(1-|k_{m,o}|^2\right)\cdot E[|f_{m-1}(n)|^2]$$
$$E[|b_m(n)|^2] = \left(1-|k_{m,o}|^2\right)\cdot E[|b_{m-1}(n-1)|^2] \quad (11)$$

As described Eq.(9), $k_{m,o}$ involves the use of ensemble averaging. Assuming that the input $u(n)$ is ergodic, we may substitute time averages for the expectation in the dominator and denominator of this equation. We thus get the reflection coefficient of stage m in the ALE[7, 8].

$$k_1 = -\frac{2\cdot b_0(1)\cdot f_0^*(2)+2\cdot b_0(2)\cdot f_0^*(3)+......}{f_0^2(2)+b_0^2(1)+f_0^2(3)+b_0^2(2)+......} = -\frac{s_0(1)\cdot s_0(2)+s_0(1)\cdot s_0(2)+s_0(2)\cdot s_0(3)+s_0(2)\cdot s_0(3)+......}{s_0^2(1)+2\cdot s_0^2(2)+2\cdot s_0^2(3)+......} \quad (12)$$

The adaptive algorithm for the estimation of input frequency is summarized as follows:

$$N(n) = \lambda\cdot N(n-1)+(1-\lambda)\cdot s(n-1)\cdot\left[s(n)+s(n-2)\right]$$
$$D(n) = \lambda\cdot D(n-1)+(1-\lambda)\cdot 2\cdot s^2(n-1) \quad (13)$$
$$k_1(n) = -\frac{N(n)}{D(n)}$$
$$k_1(n) = \begin{cases} k_1(n), & -1\le k_1(n)\le 1 \\ 1, & k_1(n)>1 \\ -1, & k_1(n)<-1 \end{cases}$$
$$\hat{k}_1(n) = \gamma\cdot \hat{k}_1(n-1)+(1-\gamma)\cdot \hat{k}_1(n)$$

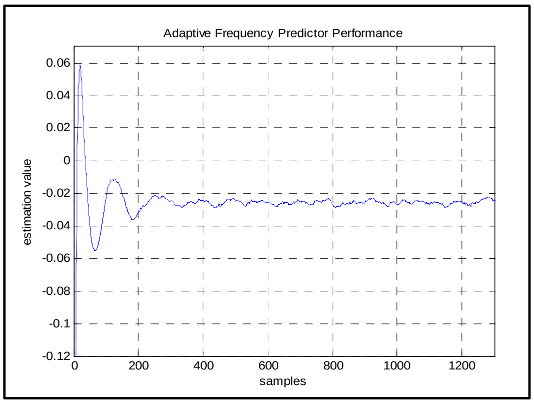

Fig. 3 The adaptive estimator performance

When the input to the ALE is a sinusoid corrupted by interference, the RLS(Recursive Least Square) algorithm in finds and tracks the frequency and consequently enhances the sine wave. In Eq.(26), $\hat{k}_1(n)$ is the estimate of the coefficient k_1 at time n. $k_1(n)$ is the intermediate value before smoothing, and $s(n)$ is the output of all-pole section. The initial values, $D(0)$ and $N(0)$ should be sufficiently small to get better transient properties. Also, λ is the forgetting factor, and γ is the smoothing factor. The coefficient is clipped in the range of [-1,1], as shown in Eq(26). Because the normalized notch frequency f_{jam} and k_1 are related in the form of $k_1 = -\cos 2\pi f_{jam}$, the frequency estimate at time n is given by

$$\hat{f}_{jam}(n) = \frac{1}{2\pi}\arccos\left(-\hat{k}_0(n)\right) \quad (14)$$

It is shown that the IIR notch filter of lattice type yields the unbiased frequency estimate regardless of the pole-zero contraction factor and the noise variance, whereas the direct form yields the biased estimate.
In Fig.6, it is presented the designed adaptive estimator performance. It can estimate the interference frequency within 200 samples and remove the interference signal within 0.19ms.

4. ADAPTIVE DOUBLE NOTCH FILTER DESIGN

In previous section, the pole-zero position is placed on unit circle because ALE is to remove interference completely. But the infinite notch depth of the IIR filter removes the information signal as well as the interference thereby causing data distortion. Hence, the self-noise that causes data distortion also needs to be removed by adjusting the optimal reverse notch filter. The ALE has been used to estimate interference center frequency and to transmitted estimated frequency parameters as Fig. 5. The proposed algorithm filter section consists of the optimal notch filter and reverse notch filter.

The optimal reverse IIR notch filter can be expressed as Eq.(15).

$$H(z) = \frac{\tau_1+\tau_2\cdot z^{-2}}{1+k_0\cdot\left(1+k_{1,inv}^2\right)\cdot z^{-1}+k_{2,inv}^2\cdot z^{-2}} \quad (15)$$

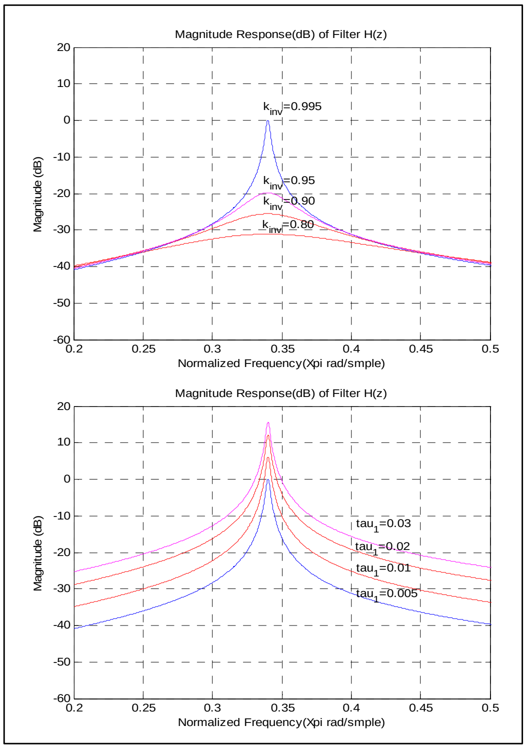

Fig. 4 The frequency response of the optimal notch filter

In Eq.(15), $k^2_{2,inv}$ and $k^2_{1,inv}$ with respect to the notch depth and width have gotten the same values to each other for exact interference estimation. Also, it is important that to decide τ_1 and τ_2 parameter values for exact estimation of the interference signal power. In Fig.(4), the closer $k_{1,inv}$ is to unity, the narrower the notch filter response (inside the notch frequency) will make the bias smaller. So the parameter τ_1 is respected to notch filter response level. Therefore, it should adjust the two parameters with consideration for interference power and bandwidth.

In this section, the adaptive double notch filter(ADNF) is designed using the optimal notch and reverse notch for suppress interference in the GPS receiver as Fig. 5. The RF/IF stage has fed received GPS signal which is quantized with 12bit into the ADNF. And the ALE of ADNF stage can estimate interference frequency using parameters of lattice notch filters. Estimated frequency can be looked up by the optimal reverse notch filter which should estimate interference signal. The output of optimal notch filter and reverse notch filter is transmitted to the input of correlator.

The proposed filter algorithm can be presented through mathematical mechanism. The received signal in the sum of signal, noise and interference has gotten the simple modeling as Eq.(16).

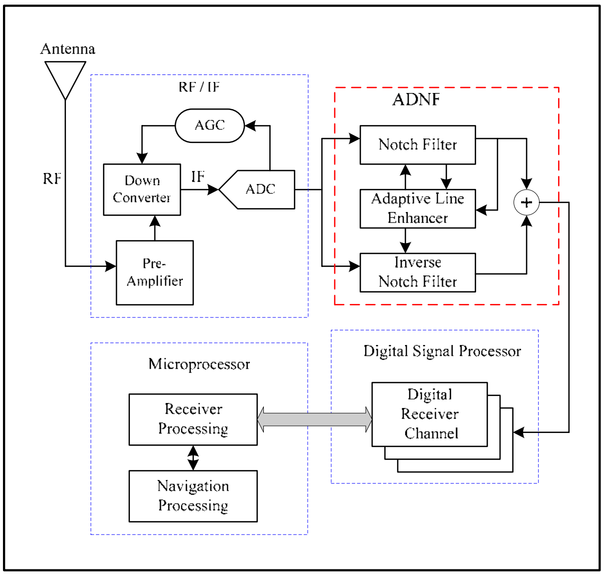

Fig. 5 The GPS receiver structure using the adaptive double notch filter

$$v(n) = d(n) + j(n) + w(n) \tag{16}$$

where $d(n)$ is the data signal multiplied by the PN code, $j(n)$ is a single tone interference represented by a sine wave with random phase, and $w(n)$ is the white Gaussian noise. If the data signal has normalized magnitude of 1 or -1 and the PN code is long enough, $d(n)$ can be considered as a sequence $p(n)$ which is either 1 or -1 with equal probability. Specifically, the input signal can be rewritten as

$$v(n) = p(n) + A\cos(w_0 n + \varphi) + w(n) \tag{17}$$

where w_0 is the center frequency of the interference, and φ is the random phase uniformly distributed over $(-\pi, \pi)$. Then the output signal of the ADNF is given by

$$y(n) = H(z)v(n) = p_{out}(n) + j_{out}(n) + w_{out}(n) \tag{18}$$

where $p_{out}(n)$, $j_{out}(n)$ and $w_{out}(n)$ are the output components of the data, interference, and Gaussian noise, respectively. $H(z)$ is the transfer function of the ADNF.

$$H(z) = H_{notch}(z) + H_{Inverse}(z) \tag{19}$$

In Eq.(19), $H_{notch}(z)$ and $H_{Inverse}(z)$ are the components of the ADNF transfer function, the transfer function of the optimal notch filter and the transfer function of the optimal reverse notch filter, respectively. Let's try to define their output into the components of ADNF.

$$\begin{aligned} y(n) &= y_{notch}(n) + y_{Inverse}(n) \\ &= H_{notch}(z)v(n) + H_{Inverse}(z)v(n) \\ &= p_{N.out}(n) + j_{N.out}(n) + w_{N.out}(n) - j_{I.out}(n) \end{aligned} \tag{20}$$

The output of the notch and the inverse notch is defined by $y_{notch}(n)$ and $y_{Inverse}(n)$. $p_{N.out}(n)$, $j_{N.out}(n)$ and $w_{N.out}(n)$ are PN code, interference and Gaussian noise at the optimal notch output. $j_{I.out}(n)$ is the estimated interference signal in the optimal reverse notch filter.

$$j_{out}(n) = j_{N.out}(n) - j_{I.out}(n) = 0 \tag{21}$$

When the optimal reverse notch filter performance is maximized, it is defined as Eq.(21). It is necessary that $j_{out}(n)$ is removed as much as possible and the pure PN code is maintained under regular condition. In order to help the ALE and reverse notch filter to estimate interference signal exactly, the adequate parameters can be used. The performance requirement of the optimal reverse notch filter is presented through the SNR of ADNF.

$$SNR_{out} = \frac{E[p^2(n)]}{E[(y(n) - p(n))^2]} \tag{22}$$

Since it is obvious that $E[p^2(n)] = 1$ in our model, $E[(y(n) - p(n))^2]$ can be obtained by deriving $E[y(n)p(n)]$ and $E[y^2(n)]$ with respect to the impulse response of the ADNF. If we assume that $p(n)$, $j(n)$ and $w(n)$ are independent from one another, $p_{out}(n)$, $j_{out}(n)$ and $w_{out}(n)$ can also be seen to be independent. Hence, it follows that

$$E[y^2(n)] = \sum_{k=0}^{\infty} h_k^2 + \sigma^2_{j_{out}} + \sigma^2_{w_{out}} \tag{23}$$

where h_k is the impulse response of the ADNF, and $\sigma^2_{j_{out}}$ and $\sigma^2_{w_{out}}$ are variances of $j_{out}(n)$ and $w_{out}(n)$, respectively. By the independence, $E[y(n)p(n)]$ is also given by

$$E[y(n)p(n)] = E[p_{out}(n)p(n)] + 0 + 0 = h_{out} \qquad (24)$$

From Eq.(22), Eq(23) and Eq(24), the output SNR is described as

$$SNR_{out} = \frac{1}{\sum_{k=0}^{\infty} h_k^2 + \sigma^2_{j_{out}} + \sigma^2_{w_{out}} - 2h_{out} + 1} \qquad (25)$$

The output SNR of ADNF is presented to have kept the derived SNR maximizing.

$$\sigma^2_{j_{out}} = E[j^2_{out}(n)] = E[(j_{notch}(n) - j_{inverse}(n))^2] \qquad (26)$$

Therefore, the output SNR varies from the output variance of interference, $\sigma^2_{j_{out}}$. Finally, the maximum output SNR is guaranteed by the estimation performance of interference in the proposed filter algorithm, ADNF.

5. SIMULATION RESULTS

In this section, computer simulation is performed to show the ADNF performance using seven different jamming scenarios as Table 1. Each narrow-band jamming is defined as any unwanted signal occupying less than the entire ±1.023MHz bandwidth of C/A code. Generally, the minimum value of 20dB JSR was chosen because C/A acquisition at 24dB JSR is a common military requirement. The maximum value 80dB was chosen because no GPS receivers can track at 80dB JSR against a wideband jammer without employing beamsteering, nulling, or some other multi-element antenna technique in the most modern weapon systems. There is simply not enough processing gain available[5].

Fig. 6 shows the effect of narrowband interference in the GPS receiver. When GPS receiver is tracking signal, it gradually increases the power of signal generator until failing to track the signal. The performance of GPS receiver are sensitive to likely In-Band interferences. In other words, In-Band interference is more influential in the GPS receiver performance than Out-Band interference which is easily removed by band-pass filter.

Using the proposed filter algorithm, ADNF, Fig. 7 shows the comparison of SNR for seven different jamming scenarios as JSR changes. It can perceive the JSR performance of GPS receiver through generating jamming signal stage by stage. In Fig. 7, the ADNF guarantees that minimum JSR performance of the receiver is more than JSR 70dB in CCW, PM, Swept CW and Frequency Hopping CW jamming scenario. The receiver which has the ADNF get loss of lock more than JSR 55dB in AM jamming scenario and to fail code tracking more than JSR 49dB in FM jamming scenario.

Fig. 8 shows that increasing rate of SNR is presented by seven different jamming scenarios. It presents difference of SNR between the processing gain and the ADNF. It is evident that performance with the ADNF enabled has almost increasing rate of SNR 50dB than the receiver without the proposed filter algorithm at JSR 74dB. The higher value of JSR, the more clear the ADNF performance as the increasing rate of SNR and anti-jamming of the receiver.

Table 1 Seven Different Jamming Scenarios (Including one without jamming)

Jamming Scenarios	Property
No Jamming	· White noise center frequency : 1575.42 MHz
Coherent CW (CCW)	· Center frequency : 1575.42 MHz
Frequency Hopping CW	· Time deviation : 10ms · Frequency deviation : 100 kHz
Swept CW (SCW)	· Center frequency : 1575.42 MHz · Frequency deviation : ±50 kHz
Amplitude Modulation (AM)	· Center frequency : 1575.42 MHz · Modulation frequency : 1 kHz · Modulation depth : 50.0 %
Frequency Modulation (FM)	· Center frequency : 1575.42 MHz · Modulation frequency : 1 kHz · Frequency deviation : ±50 kHz
Phase Modulation (PM)	· Center frequency : 1575.42 MHz · Phase repetition : 175 ns · Phase width : 175 ns

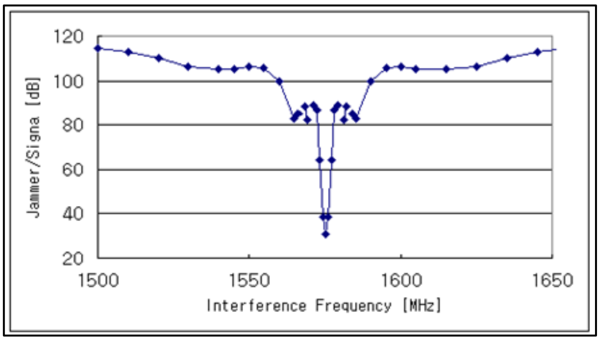

Fig. 6 J/S versus Interference Frequency at loss of lock

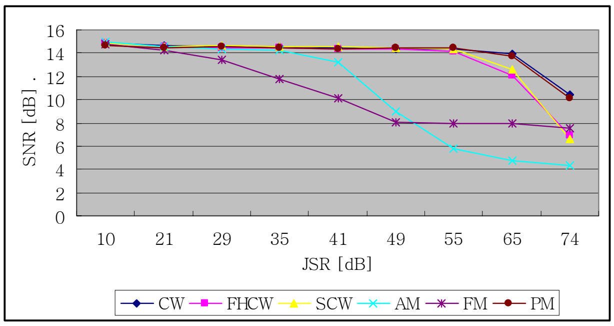

Fig. 7 SNR versus JSR for each scenario

Fig. 9 shows the comparison of before the ADNF enabled and after the ADNF enabled for each scenario. The receiver before the ADNF enabled gets an average JSR 25dB performance. However, the proposed filter algorithm guarantees that maximum JSR 79dB is presented with improved interference suppression. In CCW, Swept CW, PM and FHCW jamming scenario, the proposed filter algorithm guarantees high performance of the GPS receiver, given the GPS receivers can not track at JSR 80dB against a jammer without employing some multi-element antenna techniques. It also let receiver performance make the more than JSR 40dB in AM, FM jamming scenario. In this result, if the GPS receiver itself has anti-jamming performance more than before, anti-jamming of the ADNF performance has been improved.

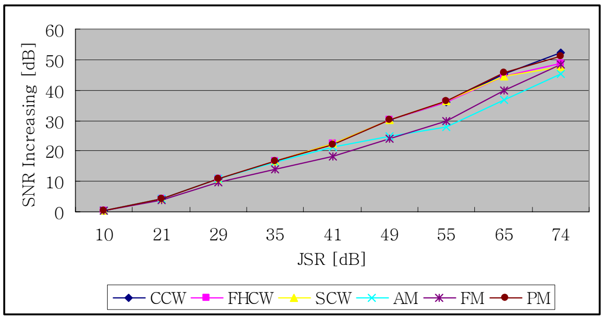

Fig. 8 ADNF performance – Increasing rate of SNR

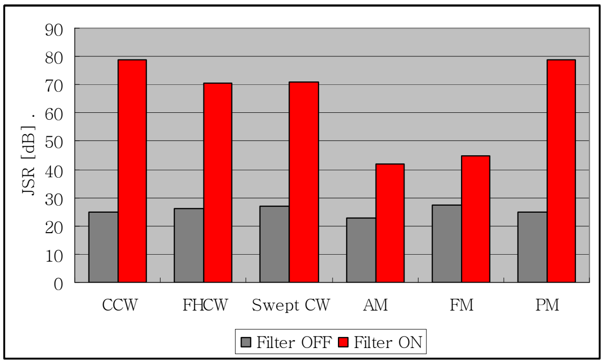

Fig. 9 Comparison of ADNF performance according to Jamming

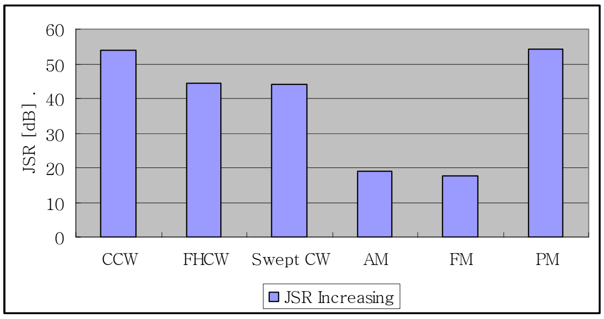

Fig. 10 ADNF performance – Increasing rate of JSR

One of EA(Electronic Attack) missions has to operate EA application during 3~8sec and be not same position more than 15 minutes for destroying and detecting the target by DF(Direction Finder). Because of this reason, FHCW and Swept CW interference signal is used primarily for EA missions. In Fig.9, the proposed filter algorithm has showed a strong anti-jamming performance in FHCW and Swept CW.

Fig. 10 also shows that the ADNF has presented increasing rate of JSR 40dB or more in FHCW and Swept CW jamming scenario. The proposed filter algorithm has kept the good performance of receiver against ECM(Electronic Counter Measures). In CCW and PM, the ADNF has guaranteed the best performance the receiver of the seven scenarios. Because it is vulnerable to CCW and PM in commercial GPS chipset receiver, the GPS receiver can minimize the effect of interference signal using the proposed filter algorithm.

6. CONCLUSION

In this paper, the adaptive double notch filter has been proposed for the excision of narrow-band interference in GPS receiver. The zeros of the notch filter are adjusted to induce the notch on the frequency of the interference using the adaptive frequency estimators. However, if the zeros are on the unit circle, the notch depth is infinite and the filter removes the information as well as the interference. This causes data distortion and the performance of the receiver is degraded below the level of the case that no excision is performed. Hence the optimal reverse notch filter is proposed to preserve data as well as interference estimation. For this purpose, we have designed the ADNF which GPS receiver structure has been aligned with parallel between the notch and reverse notch. As an estimator of the frequency, we employed the IIR adaptive line enhancer. Using the ADNF, the anti-jamming of receiver has gotten average JSR 64dB and increasing rate of JSR 38.2dB. The simulation results show that ADNF has improved anti-jamming, reliability and efficiency of the receiver.

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