

## Experimental Study for Optimizing the Acceleration of AC Servomotor Using Finite Jerk

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**Abstract:** This paper presents an experimental study for optimizing the acceleration of AC servomotor using finite jerk (the first derivative of acceleration). The acceleration optimization with finite jerk aims at generating the smooth velocity profile of AC servomotor by experimentally minimizing vibration resulted from the initial friction of servomotor. The stick-slip motion of AC servomotor induced by initial friction can result in the positional errors that are not good for high-precision devices such as the assembly robot arms to be used in a 300mm wafer or a LCD (Liquid Crystal Display) stocker system. In this paper, experiments were made by using a PM (Permanent Magnet) type AC servomotor with MMC® (Multi Motion Controller) programmed in Visual C++®. The experiments have been performed for finding the optimal duration time of finite jerk in terms of the minimization of vibration displacements when both the magnitude of velocity and the allowable acceleration are given. We have compared the proposed control with the conventional control with trapezoidal velocity profile by measuring vibration displacements. The effectiveness of the proposed control has been verified in that the experimental results showed the decrease of vibration displacement by about 24%.

**Keywords:** Experiment Study, Finite Jerk, Acceleration Optimization, Trapezoidal Velocity, MMC® (Multi Motion Controller)

### 1. INTRODUCTION

Most devices of factory automation such as NC (Numerical Control) Machines or industrial robots have been widely used for both unmanned automation and improvement of productivity. Owing to this fact, an AC servomotor, which is one of basic cells for the devices, is being rapidly developed especially for the convenience of maintenance as well as high performance.

In general, the conventional AC servomotor control has been based on a trapezoidal velocity profile as shown in Fig. 1. It is well known that this control has the advantage of easy implementation on a controller without heavy computational load or hardware complexity. But it has a hurdle in generating continuous acceleration, which can result in the infinity of jerk as shown in Fig.1. In turn, the infinite jerk can induce vibration. The vibration can be considered to result from the stick-slip motion induced by the friction of mechanical parts of a servomotor. The stick-slip motion of AC servomotor induced by initial friction can result in the positional errors that are not good for high-precision devices such as the assembly robot arms to be used in a 300mm wafer or a LCD (Liquid Crystal Display) stocker system. A conventional remedy to cope with this problem relies on the adjustment of time offsets [1] or a gain tuning approach [2-5]

A recent effort is to generate a finite jerk with half sine curve in [6]. But the approach was illustrated to be effective only in a simulation. When it is implemented on a servomotor, the burden of computation related to half sine curves can be a bottleneck of realization.

In this paper, in order to reduce the vibration effect of an AC servomotor, the infinity region of jerk profile is modified into a finite jerk with finite time duration. The servomotor-drive digital control approach based on finite jerk is constructed by using the MMC® (Multi Motion Controller), that is, the multi-axes motion controller based on a DSP (Digital Signal Processor). Experimental study on acceleration optimization of an AC servomotor will be presented to obtain the minimization of vibration by finding the optimal value of finite jerk duration, under the condition that the bound of allowable acceleration according to a commanded velocity is given.

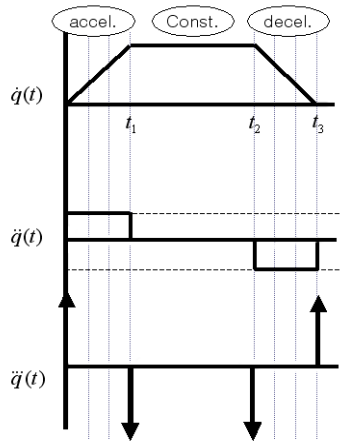


Fig.1 Acceleration and Jerk Profiles with Trapezoidal Velocity

2. FINITE JERK MOTION SYSTEM

In order to realize the stable motion of an AC servomotor system through minimizing inertial friction on the servomotor, any infinite jerk should have a finite value,  $J$ , for some time duration  $\Delta t$  as shown in Fig. 2.

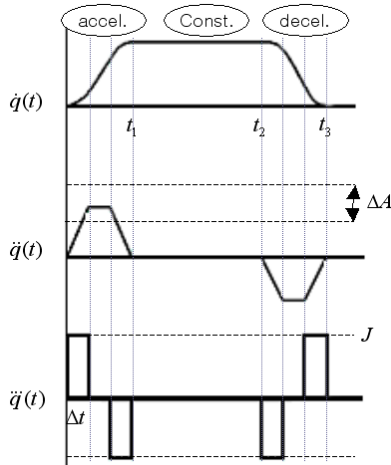


Fig. 2 Proposed Finite Jerk

So in this paper, the experimental control algorithm is made so simpler to concentrate at figuring out the improvement of finite jerk comparing to the other theories.

And gain tuning had considered already through evaluating from the analyzing BODE plot, Root-Locus plot. So the servo response is stable status.

As adjusting the finite jerk algorithm, the time periods of acceleration, constant velocity and deceleration are given as conventionally same value. And the time duration  $\Delta t$  is same again. Because we can make formulas use common variables.

And then we can only consider that how much would it be influencing to the system by changing the common duration factor,  $\Delta t$ .

In the acceleration time period, we assume that the finite jerk duration  $\Delta t$  should be bounded by Eq. (1).

$$0 < \Delta t < \frac{t_1}{2} \tag{1}$$

In Eq. (1),  $t_1$  is the total accelerating time. In the usual case of servomotor control, the magnitude of velocity,  $V$ , is given. Thus the magnitude of acceleration,  $A$ , can be obtained by,

$$A = \frac{V}{t_1 - \Delta t} \tag{2}$$

Then the jerk magnitude  $J$  comes from,

$$J = \frac{A}{\Delta t} \tag{3}$$

Based on the finite jerk, the acceleration profile  $\ddot{q}(t)$  can be derived as follows.

$$\ddot{q}(t) = \begin{cases} \frac{A}{\Delta t} t & \text{for } 0 \leq t \leq \Delta t & (4) \\ A & \text{for } \Delta t \leq t \leq t_1 - \Delta t & (5) \\ \frac{A}{\Delta t} (t_1 - t) & \text{for } t_1 - \Delta t \leq t \leq t_1 & (6) \end{cases}$$

With this acceleration profile, we have the following velocity profile  $\dot{q}(t)$  as follows.

$$\dot{q}(t) = \begin{cases} \frac{A}{2\Delta t} t^2 & \text{for } 0 \leq t \leq \Delta t & (7) \\ A \cdot t - \frac{A \cdot \Delta t}{2} & \text{for } \Delta t \leq t \leq t_1 - \Delta t & (8) \\ \frac{A}{\Delta t} (t_1 \cdot t - \frac{t^2}{2}) + -\frac{A}{2\Delta t} t_1^2 + A \cdot t_1 - A \cdot \Delta t & \text{for } t_1 - \Delta t \leq t \leq t_1 & (9) \end{cases}$$

Following the upper procedure, we can approach the other periods ( $t_2, t_3$ ).

The period of constant velocity have same time value as the acceleration time. During the period, desired maximum velocity is expressed constantly as following Eq. (10).

$$\dot{q}(t) = V_{\max} = V_{\text{desire}} \text{ for } t_1 \leq t \leq t_2 \tag{10}$$

According to the constant velocity, the acceleration, the differential calculus of velocity, is shown as Eq. (11).

$$\ddot{q}(t) = 0 \text{ for } t_1 \leq t \leq t_2 \tag{11}$$

And after that, we could find that deceleration time period also has relational formula like accelerating period.

Reminding the assumption Even though this is deceleration case, we are using the same character  $\Delta t$  and same duration value to avoid mathematical complexity.

As we considered before, assume the time duration  $\Delta t$  as following Eq.(12) for the decreasing period.

$$0 < \Delta t < \frac{t_3 - t_2}{2} = \frac{t_1}{2} \quad (12)$$

In Eq. (12),  $t_3 - t_2$  is the total decelerating time. With these condition and similar procedures done in the accelerating time period, we can lead some relative equations of the acceleration in the decelerating time period, shown Eq. (13) ~ (14).

$$\ddot{q}_i(t) = \begin{cases} -\frac{A}{\Delta t}t + \frac{A}{\Delta t}t_2 & \text{for } t_2 \leq t \leq t_2 + \Delta t & (13) \\ -A & \text{for } t_2 + \Delta t \leq t \leq t_3 - \Delta t & (14) \\ \frac{A}{\Delta t}t - \frac{A}{\Delta t}t_3 & \text{for } t_3 - \Delta t \leq t \leq t_3 & (15) \end{cases}$$

From this acceleration profile, we have drawn velocity profile,  $\dot{q}(t)$  as following Eq. (16) ~ (17)

$$\dot{q}_i(t) = \begin{cases} -\frac{A}{2\Delta t}t^2 + \frac{A}{\Delta t}T_2t + A(t_3 - t_2 - \Delta t) - \frac{A}{2\Delta t}t_3^2 & \text{for } t_2 \leq t \leq t_2 + \Delta t & (16) \\ -At + \frac{(2t_3 - \Delta t)A}{2} & \text{for } t_2 + \Delta t \leq t \leq t_3 - \Delta t & (17) \\ \frac{A}{2\Delta t}t^2 - \frac{A}{\Delta t}t_3t + \frac{A}{2\Delta t}t_3^2 & \text{for } t_3 - \Delta t \leq t \leq t_3 & (18) \end{cases}$$

As detected from upper guide, we approached both velocity and acceleration profile in the adjusted time range, we defined before.

Before the implementation, simulations of the profile were conducted before, by using the MATLAB.

Following the process of the relational formulas expressing the velocity and acceleration profile over the time range, ( $0 < t < t_3$ ), the result of the simulation is shown in Fig. 3-(a) and Fig. 3-(b). The condition of this profile is given as, same time value of each sector, ( $t_1 < t_2 - t_1 = t_3 - t_2 = 1$ ), and common time duration in both acceleration and deceleration time period ( $\Delta t = 1/3$ ).

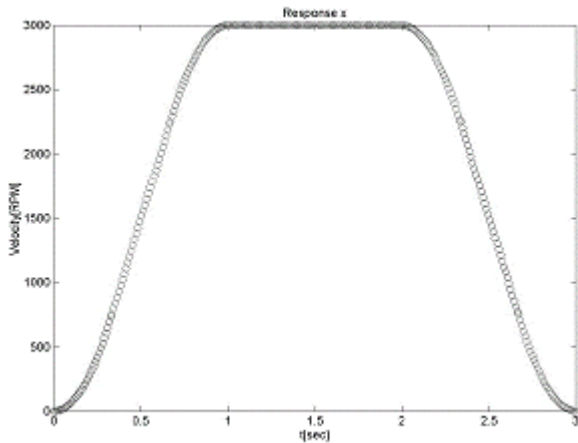


Fig. 3-(a). Simulated result profile of the velocity.

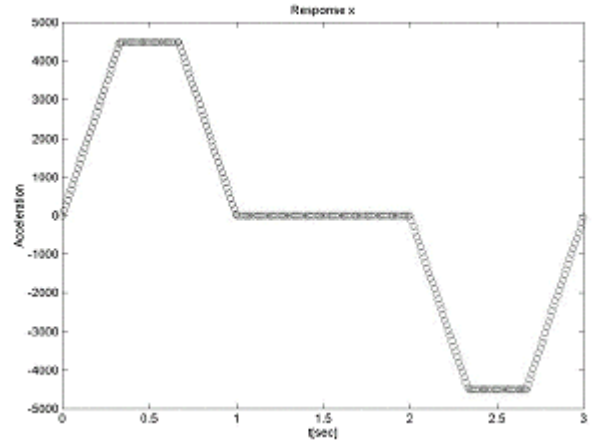


Fig. 3-(b). Simulated result profile of the acceleration.

From these profiles and formulas, real coding procedure would be conducted by using c++ under the library of the head quarter controller, MMC<sup>®</sup>.

In the real implementation of finite jerk strategy on an AC servomotor, there is a limitation on the magnitude of acceleration. In specific, when the accelerating time  $t_1$  is given to be 1 second, we can consider the extreme two cases, i. e.,  $\Delta t = t_1/2$  (S-curve velocity) and  $\Delta t = 0$  (linear velocity for trapezoidal profile), respectively, as shown in Fig. 4. Then the maximum magnitude of acceleration,  $A$ , for the finite jerk case, that is,  $0 < \Delta t < t_1/2$ , can vary between the two cases. But the value of  $A$  should be bounded by the allowable acceleration that is determined by an instantaneous maximum current. It follows that the claim of this paper is to find the optimal value of finite jerk duration,  $\Delta t$ , which minimizes vibration displacements as well as making  $A$  bounded by the allowable acceleration.

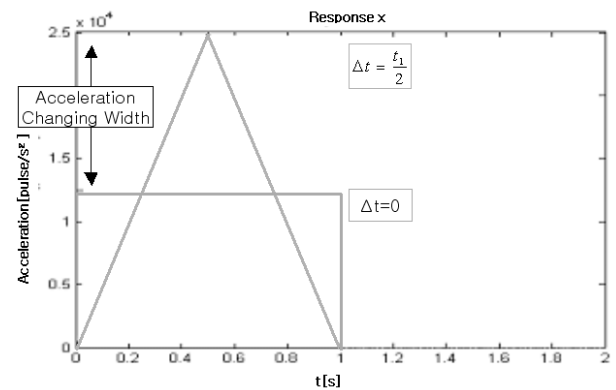


Fig. 4 Change of Acceleration according to Finite Jerk Duration

### 3. Experiment

Figure 5 shows the experimental procedure for finding the optimal value of finite jerk duration. The experiments were made by using a PM (Permanent Magnet) type AC servomotor with MMC<sup>®</sup> (Multi Motion Controller) programmed in Visual C++<sup>®</sup>. Actually the allowable acceleration is determined by

taking into account that the malfunction of servomotor should not occur at the rated (even maximum) speed due to rotor inertia. Besides, the inertial load of system has also influence upon the allowable acceleration, when the servomotor is applied to a certain system. Thus it follows that the allowable acceleration can be obtained from both the maximum torque of a specified servomotor and the characteristics of its inertial load. In fact, the allowable acceleration is a major factor on the real operation of a servomotor with an inertial load. In this experimental study, the commanded acceleration in the proposed algorithm with finite jerk is adjusted by properly selecting a value of finite jerk duration  $\Delta t$ . Finally, the optimal value of  $\Delta t$  is determined so that the minimum displacement of vibration is obtained from the results of experiments corresponding to such commanded accelerations

the allowable acceleration  $A_{allow}$  can be derived. In the experiment, when  $A$  is directly input to a servo driver without any comparison with  $A_{allow}$ , the servo driver could be downed with the error sign (E20, 22) (see Fig. 4) if  $A$  would be larger than  $A_{allow}$ . In this case, the finite jerk duration  $\Delta t$  should be reselected to meet the strong condition, i. e.,  $A < A_{allow}$ . Using the value of  $\Delta t$ , the commanded velocity based on the finite jerk algorithm can be generated.

Figure 6 shows the overall set-up of experiment including the measurement of vibration. The servomotor connected to the servo driver (CSDJ<sup>®</sup> series) and the MMC<sup>®</sup> board with the TMS320C32<sup>®</sup> of DSP is mounted on a table by a clammer. A beam-type inertial load of aluminum (length: 12cm; mass: 0.7kg) is attached to the servomotor. Especially the vibration displacement is measured by using the FFT (Fast Fourier Transform) analyzer.

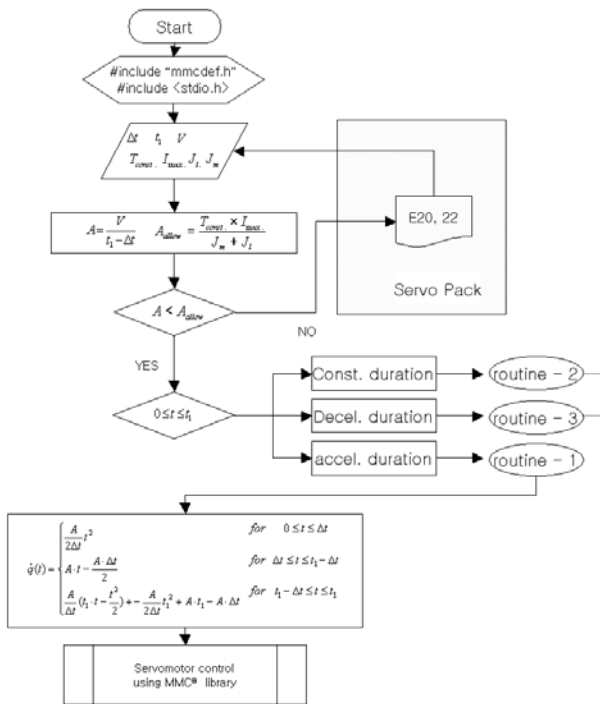


Fig. 5 Flow Chart of Experiment for Finite Jerk Algorithm

Table 1 Nomenclature of Variables used in Fig. 4

$\Delta t$	Finite Jerk Duration	$I_{max}$	Instantaneous Max. Current
$t_1$	Accelerating Time	$J_1$	Load Inertia
$T_{const.}$	Torque Constant	$J_m$	Motor Inertia

Now the flow chart shown in Fig. 5 will be explained in detail as follows. The values of the variables listed in Table 1 can be obtained from the specifications or technical manuals of an AC servomotor manufacturer. Using these values, the theoretical acceleration  $A$ , which can be given by Eq. (2), and

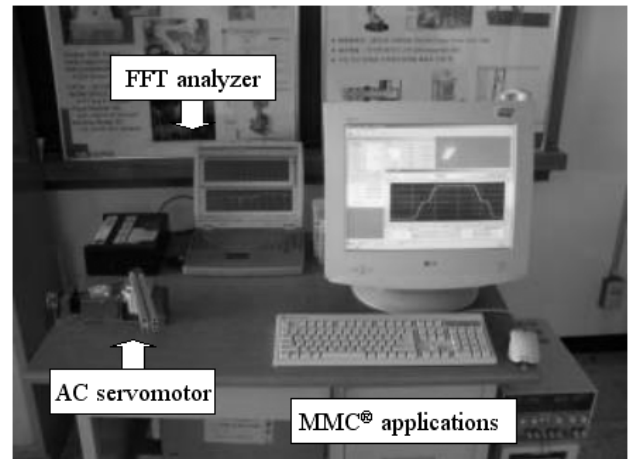


Fig. 6 Set-up of Experiment

Table 2 illustrates the specifications of the AC servomotor. The velocity motion control is based on Visual C++<sup>®</sup> by using MMC<sup>®</sup> library.

Table 2 Specifications of AC Servomotor

	Servo Motor
Output	100W
Angular Velocity	3,000 RPM
Rated torque	0.318 N · m
Weight	0.65 kg <sub>r</sub>

In the experiment for measuring vibration displacement in the conventional control with trapezoidal velocity ( $\Delta t=0$ ), the measurement of vibration has been made at 3 seconds after the servomotor begins to make rotation. This aims at synchronizing the measuring condition of every experiment. In addition, the magnitude of constant velocity is 3000RPM,

which is equal to 125,000pulse/s or 18,000deg/s that is according to the attached encoder capability(2500 pulse/revolution).

In this experiment, the accelerating time  $t_1$  is given as 1 second for the convenience of calculation. Figure 7 shows the commanded velocity with trapezoidal profile. As shown in Fig. 8, the result of vibration is shown using the FFT analyzer. Especially, the maximum acceleration is 12,500pulse/s<sup>2</sup> (see Table 3), which is less than the maximum acceleration, about 19,000pulse/s<sup>2</sup>. In addition, it is noticed that the maximum vibration displacement is 0.37mm.

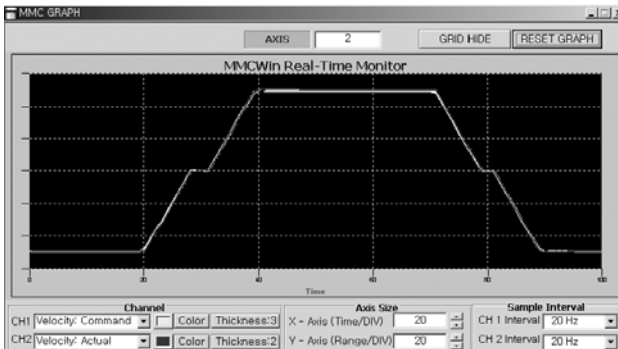


Fig. 7 Commanded Velocity with Trapezoidal Profile

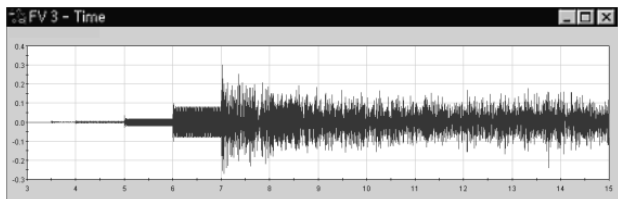


Fig. 8 Result of Vibration Displacement for Trapezoidal Velocity Profile using a FFT Analyzer

Table 3 Experimental Result of Trapezoidal Velocity

	Output Data
Condition	$\Delta t = 0$
Desired Velocity	12,500pulse/s
Accelerating Time ( $t_1$ )	10msec
Max. Vibration Displacement	0.37mm
Max. Acceleration	12,500pulse/s <sup>2</sup>

Experiments of finding the optimal value of finite jerk duration ( $\Delta t$ ) was performed in a similar manner to the case of trapezoidal velocity profile. After 10 times of experiments, we have obtained the optimal value of  $\Delta t$  as  $t_1/3$  (or  $1/3$  for the case of  $t_1=1$ ) according to the experimental procedure described in Fig. 6. Figure 9 shows the commanded velocity with the finite jerk of  $\Delta t = t_1/3$ . The result of vibration using the FFT is shown in Fig. 10. In Table 4, the maximum acceleration is 18,750pulse/s<sup>2</sup>, which is

less than the maximum acceleration. In particular, it is noticed that the maximum vibration displacement is 0.28mm, about a 24%-reduced value, compared to that of trapezoidal velocity.

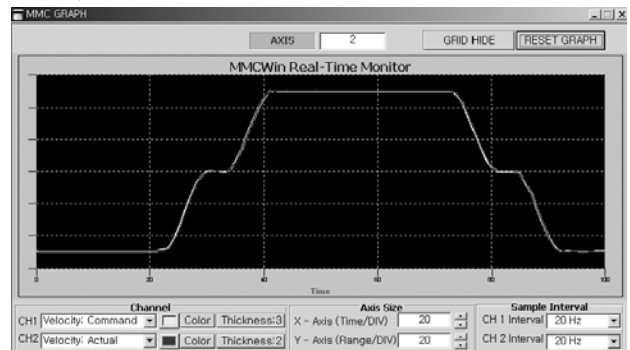


Fig. 9 Commanded Velocity with Finite Jerk ( $\Delta t = t_1/3$ )

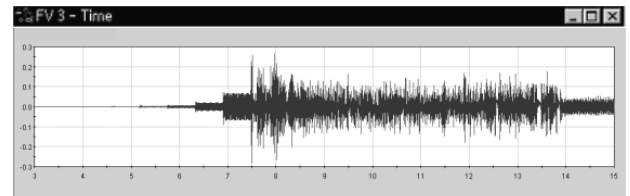


Fig. 10 Result of Vibration Displacement for Finite Jerk ( $\Delta t = t_1/3$ ) using a FFT Analyzer

Table 4 Experimental Result of Finite Jerk ( $\Delta t = t_1/3$ )

	Output Data
Condition	$\Delta t = t_1/3$
Desired Velocity	12,500pulse/s
Accelerating Time ( $t_1$ )	10msec
Max. Vibration Displacement	0.28mm
Max. Acceleration	18,750pulse/s <sup>2</sup>

#### 4. CONCLUSION

This paper has presented an experimental study on AC servomotor acceleration optimization using finite jerk, which aims at generating the smooth velocity profile of AC servomotor by experimentally minimizing vibration resulted from the initial friction of servomotor. The experiments were made by using a PM type AC servomotor with MMC® programmed in Visual C++®. Experiments have been performed for finding the optimal duration time of finite jerk in terms of the minimization of vibration displacements when both the magnitude of velocity and the maximum allowable acceleration are given. The comparison of the proposed velocity control with the conventional control with trapezoidal velocity profile has made by measuring vibration displacements. The effectiveness of the proposed control has been verified in that the experimental results showed the decrease of vibration peak by about 24%. The application of the proposed acceleration optimization with finite jerk will be expecting for high-precision devices such as the assembly robot arms to be used in a 300mm wafer or a LCD stocker system.

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**ACKNOWLEDGMENTS**

This work was partially supported by the Korea Science and Engineering Foundation (KOSEF) through the Machine Tool Research Center at Changwon National University and Robot Valley Co., Ltd. This work was also partially supported by Knowledge-based MEchanical parts and Materials (K-MEM) R&D Cluster at Changwon National University. This work was also partially supported by the Ministry of Commerce, Industry and Energy (MCIE) through the program for cultivating graduate students in regional strategic industry.

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