

Extension and Simplification of Inverse LQ Regulator of Large Scale Systems by Decentralized Control

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Abstract: An LMI based method to construct a decentralized control law for large scale systems is discussed. It is extended to assure the stability not only of the overall system but also of each subsystem without interconnection. Then, it is simplified to have local feedback loops only for some selected subsystems.

Keywords: large scale system, decentralized control, linear quadratic regulator, linear matrix inequality

1. Introduction

Decentralized control is known to be a practical way to stabilize a large scale system which consists of multiple subsystems. Geromel and Bernussou proposed a method to obtain the optimal decentralized control via the mathematical programming[1], [2]. Ikeda and Šiljak constructed a linear quadratic (LQ) regulator of each subsystem, and modified the parameters to make the overall closed loop system an LQ regulator[3], [4].

Recently, the present author proposed a method to construct a decentralized control law for large scale systems[5]. The feedback gain was calculated with a solution of linear matrix inequalities (LMIs). The resulting closed loop system belonged to a class of LQ regulators, so it was assured to have the good robustness properties [6], [7]. The method proposed in [5] assured only the stability of the overall closed loop system.

In this paper, firstly, an extended method, which assures the stability not only of the overall system but also of each subsystem without interconnection, of the method presented in [5] is proposed. The LMIs with some extra inequalities corresponding to the stability of each subsystem are solved to obtain the decentralized feedback gain. Next, the simplification of the control structure is considered. In the method in [5], all subsystems were treated equally, but in practice, it is not necessary to control all subsystems. So, a method to reduce the number of feedback loops is proposed. In this method, only some selected subsystems are controlled. Subsystems not to be controlled are assumed to have no input channels, that is, the input matrices are forced to be zero matrices, and decentralized feedback gain is calculated. If the decentralized control stabilizing the overall closed loop system does exist, then it means that it is not necessary to control the subsystems of zero input matrices. So subsystems are divided into two groups: one group to be controlled and the other group not to be controlled. There may be various combinations of controlled and not controlled groups. One way to find the optimal set among them is to introduce another cost function, which is the summation of the costs defined to each decentralized controller. For some of the selections, the decentralized controller may not be found. For the other of the selections, it may be found. Among the latter selections, we should choose the set of controlled subsystems

to minimize the newly introduced cost function. Even some of the subsystems are not controlled, the resulting overall closed loop system is still stable and belongs to a class of LQ regulators, so it is assured to have good robustness properties.

2. Preliminary Results

Let us consider a large scale system consists of N interconnected subsystems described by

$$\dot{x}_i(t) = \sum_{j=1}^N A_{ij}x_j(t) + B_i u_i(t) \quad (i = 1, 2, \dots, N) \quad (1)$$

$$x_i(0) = x_{i0} \quad (i = 1, 2, \dots, N)$$

where $x_i(t) \in \mathbf{R}^{n_i}$ is the state of the i-th subsystem, $u_i(t) \in \mathbf{R}^{m_i}$ is the input of the i-th subsystem, and $A_{ij} \in \mathbf{R}^{n_i \times n_j}$, $B_i \in \mathbf{R}^{n_i \times m_i}$ are constant matrices. $A_{ij}(i \neq j)$ denotes the interconnection from the j-th subsystem to the i-th subsystem. x_{i0} is the initial state of the i-th subsystem. The overall system consists of subsystems (1) can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$x(0) = x_0$$

where

$$x(t) = [x_1^T(t) \quad x_2^T(t) \quad \dots \quad x_N^T(t)]^T,$$

$$u(t) = [u_1^T(t) \quad u_2^T(t) \quad \dots \quad u_N^T(t)]^T,$$

$$x_0 = [x_{10}^T \quad x_{20}^T \quad \dots \quad x_{N0}^T]^T,$$

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix},$$

$$B = \text{blockdiag} (B_1, B_2, \dots, B_N).$$

The plant parameters are assumed to satisfy the following condition.

Condition 1: There exist symmetric matrices $S_i \in \mathbf{R}^{n_i \times n_i}$ ($i = 1, 2, \dots, N$) and $T_i \in \mathbf{R}^{m_i \times m_i}$ ($i =$

4.2. Grouping of Controlled Subsystems

If the i -th subsystem belongs to S_r ,

$$\tilde{B}_i = 0 \quad (47)$$

means

$$u_i(t) = 0, \quad (48)$$

that is, the i -th subsystem is not needed to be controlled. There may be various combinations of controlled and not controlled groups. One way to find the optimal set among them is to introduce another cost functional

$$C = \sum_i c_i \quad (49)$$

where c_i is the cost of the decentralized controller for the i -th subsystem.

For some of the selections, the decentralized controller may not be found. For the other of the selections, it may be found. Among the latter selections, we should choose the set of controlled subsystems to minimize the newly introduced cost function. Even if some of the subsystems are not controlled, the resulting overall closed loop system is still stable and belongs to a class of LQ regulators, so it is assured to have good robustness properties.

5. Conclusion

An LMI based method to construct an optimal decentralized feedback law for large scale systems was discussed. It was extended to assure the stability not only of the overall system but also of the subsystems without interconnections. It was also simplified so that not all but only some of the subsystems were needed to be controlled.

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