Vibration Suppression Control for a Geared Mechanical System: Simulation Study on Vibration Suppression Effects Using a Model-Based Control with a Rotational Speed Sensor

Masahiko ITOH*

* Department of Mechanical Engineering, Miyagi National College of Technology, Natori-shi, Japan
(Tel : +81-22-381-0316; E-mail: itoh@miyagi-ct.ac.jp)

Abstract: This paper deals with a control technique of eliminating the transient vibration of a geared mechanical system. This technique is based on a model-based control with a rotational speed sensor in order to establish the damping effect at the driven machine part. A rotational speed sensor is installed in a driven gear, namely a bull gear. A control model is composed of a reduced-order mechanical part expressed as a transfer function between the rotational speed of the motor and that of the bull gear. This control model estimates a load speed after the rotational speed of the bull gear is acted on the transfer function. The difference between the estimated load speed and the motor speed is calculated dynamically and it is added to the velocity command to suppress the transient vibration generated at the load. This control technique is applied to a dies driving spindle of a form rolling machine. In this paper, the performance of this control method is examined by simulations. The settling time of the residual vibration generated at the loading inertia can be shortened down to about 1/2 of the uncompensated vibration level.

Keywords: Geared System, Form Rolling Machine, Transient Vibration, Model-Based Control, Reduced-Order Model, Rotational Speed Sensor.

1. INTRODUCTION

In the field of plastic processing machines, such as form rolling machines and thread rolling machines, a tool is often connected to the driving part composed of a motor and a gear reducer through a coupling and a shaft. However, owing to the existence of the backlash in the gear train and the natural frequency of the mechanical system, several kinds of vibration phenomena are induced in these servo systems. For example, transient vibrations related to the backlash and the mechanical eigenvalues in starting and arrival phases can be taken up. The occurrence of the transient vibration causes problems such that the tact time of the system may be lengthened and the processing accuracy of the products may be deteriorated.

For the single-drive geared system with backlash, several kinds of control methods are proposed and their effectiveness is investigated in many papers. For instance, the full-closed loop control using sensors at the end-effectors [1], a torque compensation control using the PD feedback loop with a disturbance observer [2] and a speed control method using the gear torque observer and a feedback gain [3] are well known.

This paper deals with a control technique to suppress the transient vibration mainly related to the natural frequency of the mechanical system and the backlash. As the author had proposed easily realizable model-based control described in [4], a new model-based technique is proposed here. This technique is based on an easily realizable control technique using a model-based control with a rotational speed sensor attached to the gear reducer's output shaft. In referring to the new model-based control technique, the reduced-order control model as a dynamical compensator can be easily obtained as a transfer function between the rotational speed of the load and that of the gear reducer's output shaft. The transfer function does not depend on the existence of the tooth separation at the gear stage. This control model calculates the rotational speed of the driven mechanical part after the rotational speed of the bull gear is acted on the transfer function. The difference between the estimated rotational speed of the driven machine part and the motor speed is calculated dynamically, and it is added to the velocity command to suppress the transient vibration after being multiplied by a gain. The function of this technique is to establish a damping effect at the driven mechanical part.

This control technique is applied to the geared mechanical system composed of a servo motors, spur gears and a loading inertia connected to the bull gear through a torsion-bar, which is assumed to be a dies driving spindle. The reduction ratio of the gear stage is 18/100. The first natural frequency of the mechanical system with neglecting the backlash is about 28 Hz. The amount of backlash is set to 0.085mm according to JIS B 1703.

Here, the effectiveness of the proposed control technique is examined by simulations on stability, frequency and time responses. The simulations show satisfactory control results in reducing the transient vibration of the geared mechanical system with backlash. As a result, the settling time of the transient vibration related to the first vibration mode can be shortened down to about 1/2 of the uncompensated vibration level.

2. SCHEMA OF FORM ROLLING MACHINE

Figure1 shows a schematic diagram of a form rolling machine, which is one of the plastic processing machines. It consists of right and left spindles for driving the dies, dies, gears and servo motors. Here, servomechanisms for the spindle tilt control and the distance between right and left spindles control are not mentioned. It products ball screws, helical gears, worm gears, etc.

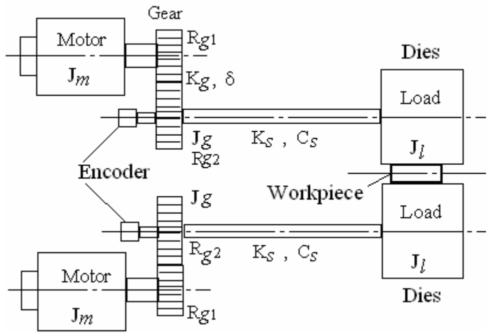


Fig. 1 Schematic diagram of a form rolling machine.

3. MODELING OF A DIES DRIVING SPINDLE

3.1 Schema of a dies driving spindle

As a controlled system, a dies driving servo mechanism is taken up as shown in Fig.2 in this paper. Because the development of a control scheme to reduce the transient vibration generated in the dies driving spindle should be considered the most important theme in the form rolling machine.

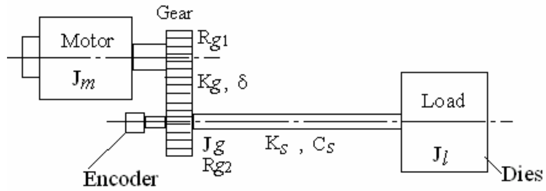


Fig. 2 Schematic diagram of a dies driving spindle.

3.2 Modeling of backlash

The backlash between the pinion and the bull gears induces the transmission-delay of the motor torque, when the tooth separation occurs. Therefore, the backlash can be modeled into the delay element as shown in Fig.3.

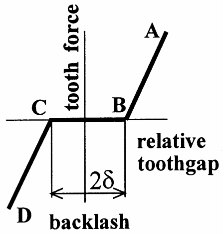


Fig. 3 Modeling of backlash.

3.3 Equations of motion

In general, as a pinion gear is rigidly connected to a motor shaft, this dies driving system can be regarded as a 3-mass system with considering the torsional stiffness of the mechanical system. Further, this system is often controlled by the velocity control using the PI action.

With neglecting the damping coefficient at the meshing point of the gear stage, equations of motion of this geared system are written in Eq.(1), when the tooth separation between the pinion and the bull gears does not occur.

$$\begin{aligned} &J_m\ddot{\theta}_m + K_g R_{g1}(R_{g1}\theta_m - R_{g2}\theta_g) = T_m, \\ &J_g\ddot{\theta}_g + K_g R_{g2}(R_{g2}\theta_g - R_{g1}\theta_m) \\ &\qquad + C_s(\dot{\theta}_g - \dot{\theta}_l) + K_s(\theta_g - \theta_l) = 0, \\ &J_l\ddot{\theta}_l + C_s(\dot{\theta}_l - \dot{\theta}_g) + K_s(\theta_l - \theta_g) = 0. \end{aligned} \tag{1}$$

On the other hand, when the tooth separation occurs at the meshing points, equations of motion can be written as

$$\begin{aligned} &J_m\ddot{\theta}_{m1} = T_m, \\ &J_g\ddot{\theta}_g + C_s(\dot{\theta}_g - \dot{\theta}_l) + K_s(\theta_g - \theta_l) = 0, \\ &J_l\ddot{\theta}_l + C_s(\dot{\theta}_l - \dot{\theta}_g) + K_s(\theta_l - \theta_g) = 0, \end{aligned} \tag{2}$$

where

θ_m = angular rotation of the motor,
θ_g = angular rotation of the bull gear,
θ_l = angular rotation of the driven machine part,
T_m = output torque of the motor,
J_m = moment of inertia of the motor including the pinion,
J_g = moment of inertia of the bull gear,
J_l = moment of inertia of the driven machine part,
R_{g1}, R_{g2} = pitch radiuses of the pinion and the bull gears,
K_g = tooth stiffness of the gear pair,
K_s = torsional stiffness between the bull gear and the loading inertia,
C_s = damping factor between the bull gear and the loading inertia,
2δ = the amount of backlash.

Further, when the motor speed is controlled by the PI action, equations related to the motor armature are expressed as

$$L\frac{di}{dt} = K_c(K_v e + \frac{K_v}{T_i}\int edt - K_{cb}i) - Ri - K_e\omega_m, \tag{3}$$

$$e = \omega_{cmd} - \omega_m.$$

The output torque of the motor is expressed as

$$T_m = K_t i, \tag{4}$$

where

ω_{cmd} = velocity command,
ω_m = rotating speed of the motor,
ω_g = rotating speed of the bull gear,
ω_l = rotating speed of the driven machine part,
e = error,
i = current of the armature,
R = motor armature resistance,
L = motor armature inductance,
K_t = torque constant,
K_e = voltage constant,
K_c = current loop gain,
K_{cb} = current feedback gain,
K_v = proportional gain of the PI control,
T_i = integral time constant of the PI control.

According to Eqs.(1)~(4), a block diagram of the control system for driving the dies spindle can be expressed as Fig.4.

4. CONTROL ELEMENTS

4.1 Installation of a speed sensor in the driven gear

The toque transmission system of the dies driving spindle is usually composed of a geared motor, a coupling, a universal joint, a shaft and a dies. These machine elements often induce torque ripples and velocity variations in the driven machine part. To compensate these ripples, a velocity sensor attached to the bull gear is useful. Here, the velocity sensor such as an encoder with an F/V converter is installed in the driven gear as shown in Fig.2 to estimate the rotating velocity of the load.

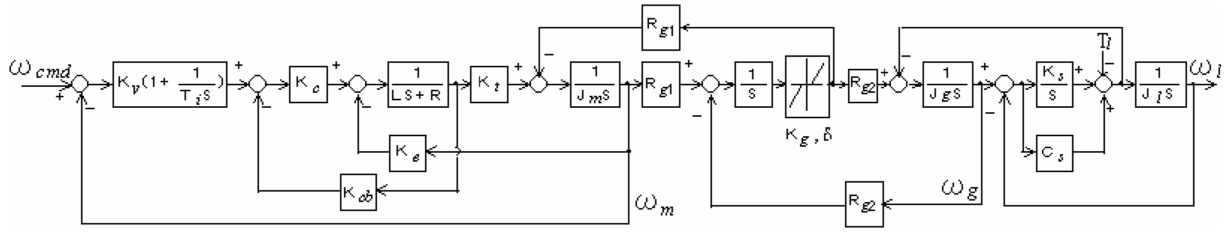


Fig.4 Block diagram of a dies driving servo mechanism.

4.2 Reduced-order control model

In referring to the dies driving system, a shaft connects a dies to a coupling or a universal joint. In case the torsional stiffness of the shaft is weak, the transient vibrations are induced at the dies when the motor starts or stops. As only the feedback of the velocity of the bull gear cannot suppress the transient vibrations, the velocity of the dies should be estimated by using the reduced-order model of the driven machine part.

This paper deals with a case such that the residual vibration is mainly dominated by the first vibration mode and the higher order vibration modes are apart from the first one.

In case that the tooth separation des not occur at the gear stage, the 3-mass system of the mechanical part shown in Fig.4 is transformed into a 2-mass system shown in Fig.5 by considering that the torsional stiffness of the gear stage is much higher than that of the shaft. In this linear reduced-order model, the equations of motion are expressed as

$$J'_g \ddot{\theta}_g + C_s(\dot{\theta}_g - \dot{\theta}_l) + K_s(\theta_g - \theta_l) = T_g \, , \tag{5}$$

$$J_l \ddot{\theta}_l + C_s(\dot{\theta}_l - \dot{\theta}_g) + K_s(\theta_l - \theta_g) = -T_l \, , \tag{6}$$

where

$$J'_g = \left\{ J_m \left(\frac{R_{g2}}{R_{g1}} \right)^2 + J_g \right\}, \tag{7}$$

T_g and T_l represent the torque of the driving part and the loading torque, respectively, as shown in Fig.6.

By using the Laplace transform on Eqs. (5), (6), the transfer function ω_l / ω_g can be obtained as

$$\frac{\omega_l}{\omega_g} = \frac{s\theta_l}{s\theta_g} = \frac{s\theta_l / T_l}{s\theta_g / T_l} = \frac{sC_s + K_s}{s^2 J_l + sC_s + K_s} \equiv G(s). \tag{8}$$

In case that the tooth separation occurs at the gear stage, the transfer function ω_l / ω_g can be also obtained as Eq.(8) with setting $T_g = 0$ and $J'_g = J_g$.

4.3 Estimation of rotating velocity of the loading inertia

To estimate the rotational velocity of the load, namely the dies, the measured rotational velocity of the bull gear is acted on the transfer function $G(s)$ as shown in Fig.7.

5. MODEL-BASED CONTROL SYSTEM

Figure8 shows a block diagram of a model-based control system with a rotational velocity sensor. In the compensating control loop, first, the estimated load speed ω'_l is converted at the motor shaft after being multiplied by R_{g2}/R_{g1}. Second, the difference between the estimated load speed and the motor speed ω_m is dynamically calculated, and it is multiplied by the gain K_b. Finally, $K_b(\omega'_l - \omega_m)$ is added to the velocity command ω_{cmd} as

$$\omega'_{cmd}(t) = \omega_{cmd}(t) + K_b \left\{ \left(\frac{R_{g2}}{R_{g1}} \right) \omega'_l(t) - \omega_m(t) \right\}. \tag{9}$$

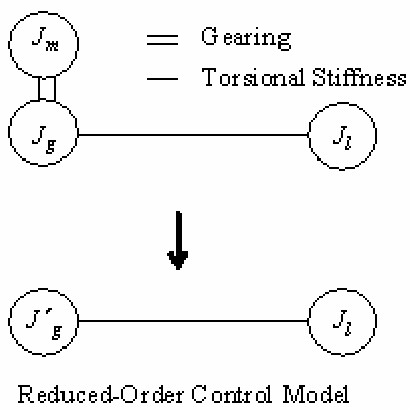


Fig.5 Transform of a 3-mass system to a 2-mass system.

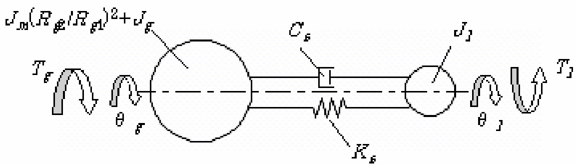


Fig.6 Modeling of a 2-mass system.

Fig.7 Relationship between ω_g and ω_l.

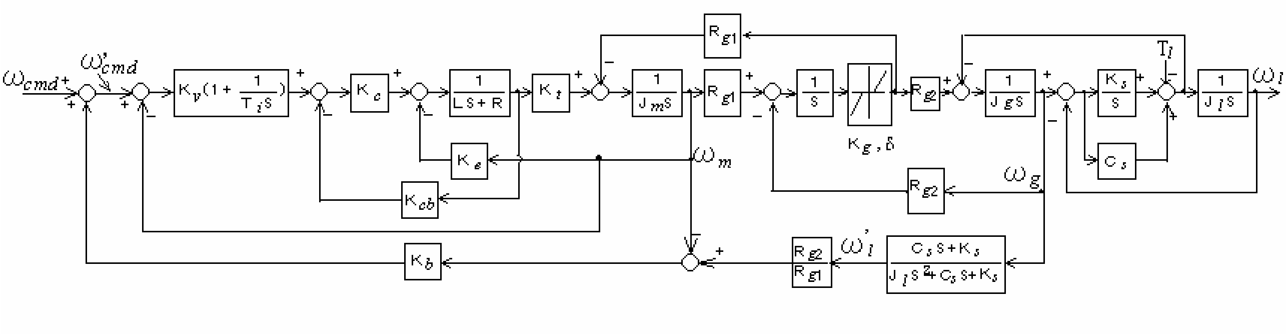
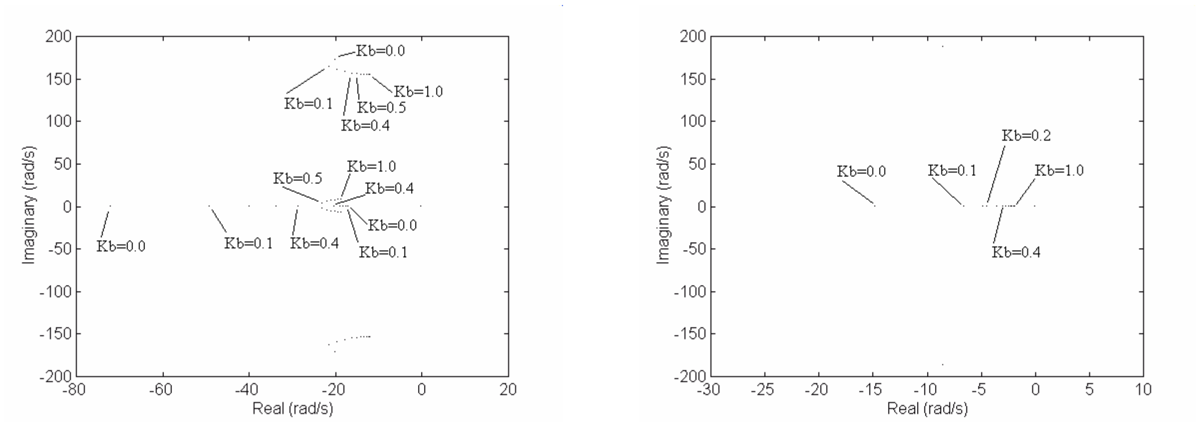


Fig.8 Block diagram of a model-based control system with a rotational speed sensor for a dies driving spindle.



a) Without tooth separation at the gear stage. b) With tooth separation at the gear stage.

Fig.9 Loci of the system eigenvalues when using the model-based control with a speed sensor.

Table 1 Simulation conditions.

Parameter		Value	Unit
Moment of inertia	J_m	1.572×10^{-5}	kg·m^2
	J_g	1.054×10^{-3}	
	J_l	5.160×10^{-4}	
Tooth stiffness	K_g	$135.6\times10^{+6}$	N/m
Torsional stiffness	K_s	12.173	N·m/rad
Damping coefficient	C_s	0.0075	N·m·s/rad
Gear reducer			
Reduction ratio	R_g	5.56	Z_2/Z_1=100/18
Module	m	1.0	
Amount of backlash	2δ	85	μm
Velocity loop gain	K_v	0.015	A/(rad/s)
Integral time constant	T_i	0.06	S
Torque constant	K_t	0.316	N·m/A
Voltage constant	K_e	0.316	V/(rad/s)
Phase resistance	R	4.5	Ω
Phase inductance	L	0.019	H
Current loop gain	K_c	118.84	V/A
Current feedback gain	K_{cb}	1.0	–
Feedback gain	K_b	0.0 or 0.7	–

6. STABILITY

The stability is considered by calculating the loci of system eigenvalues on the condition that the tooth separation occurs or not, with changing the value of K_b from 0.0 to 1.0. Though this approach does not strictly guarantee the stability of the nonlinear system, it is useful in the design phases of the system. Table 1 shows conditions for simulations. The linear first natural frequency of the geared mechanical system is 27.6 Hz and the damping ratio is 0.115.

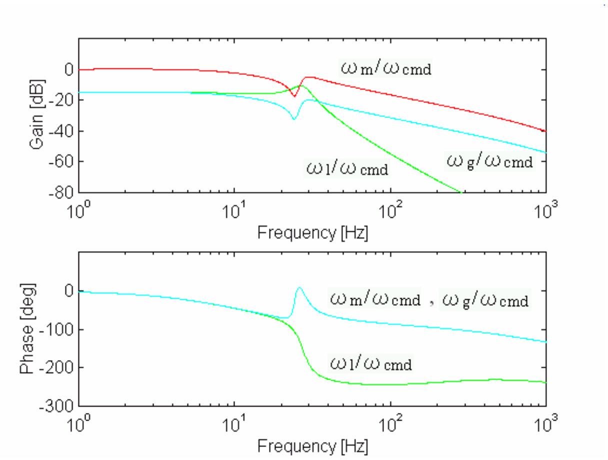
Figure9 shows the loci of system eigenvalues which exist near the imaginary axis. The eigenvalues of about -8.4 ±187.5j in Fig.9 b) are related to the natural frequency of the torsional vibration. Here, j represents the imaginary unit. Further, the eigenvalue related to the torsional vibration varies with depending on the value of K_b when the tooth separation does not occur as shown in Fig.9 a). Figure9 indicates that the control system is stable within the conditions of K_b in the case that the tooth separation occurs or does not occur.

7. SIMULATION OF FREQUENCY RESPONSE

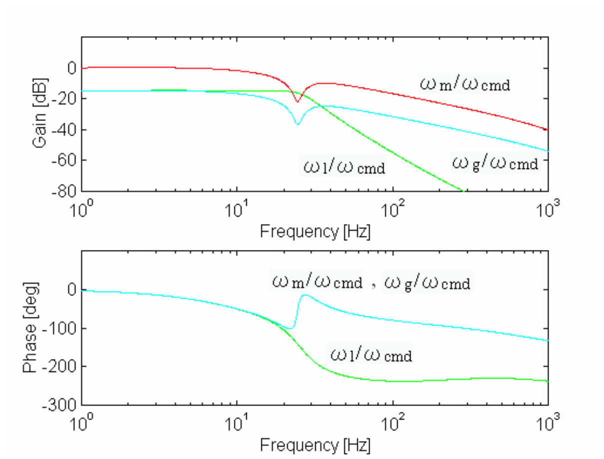
Figure10 shows the bode plots of the transfer function ω_m / ω_{cmd}, ω_g / ω_{cmd} and ω_l / ω_{cmd}. The simulation conditions are shown in Table 1. Here, the amount of backlash is set to 0.0.

In these simulations, K_b is set to 0.7 with considering the stability condition and the suppression effect on the vibration when the proposed control technique is used.

Making a comparison between Fig.10 a) and b) with paying attention to ω_l / ω_{cmd}, the proposed model-based control with a rotational speed sensor can suppress the peak of ω_l / ω_{cmd} by -6.2dB, namely 1/2 of the uncompensated level.

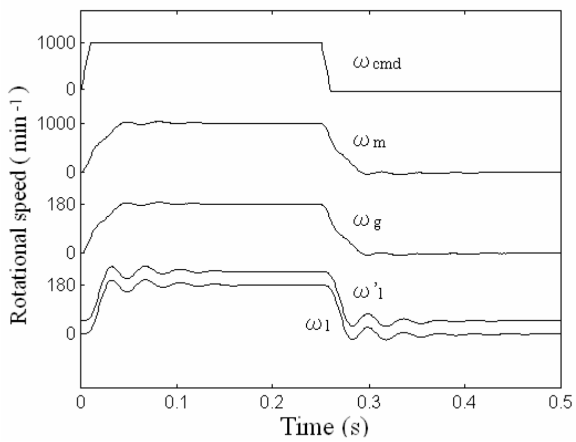


a) Without the model-based control with a sensor (K_b=0.0).

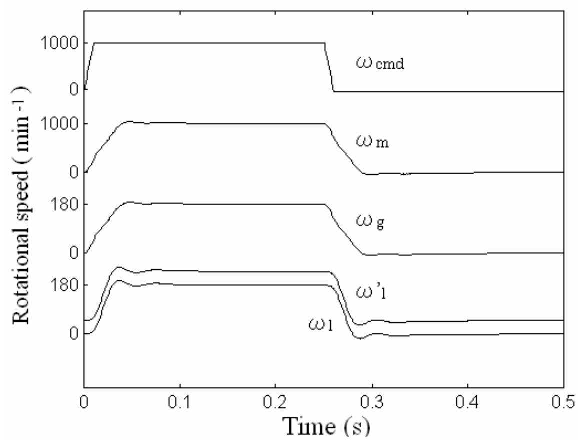


b) With the model-based control with a sensor (K_b=0.7).

Fig.10 Simulation results of frequency responses.



a) Without the model-based control with a sensor (K_b=0.0).



b) With the model-based control with a sensor (K_b=0.7).

Fig.11 Simulation results of time responses.

8. SIMULATION OF TRANSIENT VIBRATION

The step response is calculated in order to verify the suppression effects on the transient vibrations. Simulation conditions are shown in Table 1. The amount of backlash 2δ is set to 0.085mm according to JIS B 1703.

In simulations, a trapezoidal velocity profile is assigned. The constant acceleration in the starting phase is 1000min^{-1}/10ms. The cruise velocity is 1000min^{-1}. The constant deceleration in the arrival phase is -1000min^{-1}/10ms. The value of K_b is set to 0.7 according to the simulation results of frequency responses.

Figure11 shows simulation results. The rotational speeds of command ω_{cmd}, motor rotor ω_m, bull gear ω_g, driven machine part ω_l and the estimated load speed ω'_l are shown in these figures.

Making a comparison between Fig.11 a) and b) with paying attention to ω_l, the proposed model-based control with a rotational velocity sensor suppresses the residual vibration of the driven machine part. The settling time of ω_l can be reduced down to about 1/2 of the uncompensated level (from 76ms to 43ms).

9. CONCLUSIONS

A model-based control with a rotational speed sensor is proposed to eliminate the transient vibration generated in a dies driving spindle for a form rolling machine that is one of the plastic processing machines. The control model of dynamical compensator is composed of a linear reduced-order mechanical model and can be easily obtained as a transfer function between the rotational speed of the load and that of the gear reducer's output shaft. The transfer function does not depend on the existence of the tooth separation at the gear stage.

This control model calculates the rotational speed of the driven mechanical part after the rotational speed of the bull gear is acted on the transfer function. The difference between the estimated rotational speed of the driven machine part and the motor speed is calculated dynamically, and it is added to the velocity command to suppress the transient vibration after being multiplied by a gain.

The effectiveness of this control method was verified by the simulations. The simulations on the frequency and the time responses showed satisfactory control results in reducing the vibration of the driven machine part, namely the dies. As a result, the settling times in the starting and the arrival phases are reduced down to about 1/2 of the uncompensated vibration level.

REFERENCES

[1]Futami, S. et al., "A Collision-Free Control Method of Servomechanism with Backlash Using Microcomputer," *Precision Machinery*, (in Japanese), Vol. 50, No. 11, pp. 1791-1797, 1984.

[2]Wu, Y. et al., "Vibration Suppression Control for Motor Drive System with Torsional Shaft and Backlash," *Trans. SICE*, (in Japanese), Vol.34, No. 11, pp.1639-1644, 1998.

[3]Odai, M. et al., "Speed Control of 2-Inertia System with Gear Backlash Based on Gear Torque Compensation," *Trans.IEE Japan*, (in Japanese), Vol.120(D), No.1, pp.5-10, 2000.

[4]Itoh, M., "Suppression of Transient Vibration for Geared Mechanical System with Backlash Using Model-Based Control," *JSME International Journal*, Vol.47(C), No.1, pp.327-334, 2004.