Chattering Alleviation using Integral Sliding Mode Control (ICCAS 2005)

Tae Won Kim*, Min Chan Kim**, Seung Kyu Park**, Ho Gyun Ahn**
*Dept. of Electrical Engineering Changwon Polytechnic College
#110 Jungang-dong, Changwon city, Kyungnam, 641-772, Korea
**Dept. of Electrical Engineering Changwon National University
#9 sarim-dong Changwon city, Kyungnam, 641-773, Korea
(Tel: +82-55-279-7514; e-mail: skpark@changwon.ac.kr)

Abstract: The input chattering in the sliding mode control (SMC) is alleviated through a low pass filter. When the low pass filter is added to the original system, the overall system including the low pass filter dynamics can not satisfy the matching condition. So the integral SMC is applied for a main controller. A sliding surfaces are designed carefully to make the overall dynamics same with the nominal control system.

Keywords: Integral Sliding Mode Control, Chattering Alleviation, Robust Control

1. INTRODUCTION

Sliding mode control has robustness for uncertainties. It can keep the dynamics of the controlled system on the sliding surfaces which have the desired dynamics. The classical SMC require matching condition for uncertainties and its inputs show chattering. SMC is not robust during the reaching phase. These have been major drawbacks of the SMC.[1][2]. The chattering problem can be solved by using sigmoid function instead of signum function[3]. But they have some error in its performance. Observer is also used to solve this problem[4]. The matching condition also can be eliminated by some results. Significant development has been made by the integral sliding mode control in solving these problems[5][6]. It does not have reaching phase and have the desired dynamics which are controlled by other control theories. This means that the robustness of SMC can be added to other control theories. The still existing problem in integral SMC is a chattering problem. In this paper, input chattering is alleviated through a low pass filter. When the low pass filter is added to the original system, the overall system doesn’t satisfy matching condition at all. Integral sliding mode control is capable of this unmatched system. Now the job is to design the integral sliding mode control including low pass filter. When considering this problem, the first issue is how to determine the nominal control input.

2. PROBLEM FORMULATION

Consider the uncertain system of the form

\[ x(t) = Ax(t) + Bu(t) + h(t) \]

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R}^m \) is the control input and \( ||h(t)||<\rho \) represents uncertainties without matching condition.

The integral SMC is used in this paper. Integral sliding mode control makes the states stay on the following sliding surfaces.

\[ S(t) = x(t) - z(t) = 0 \]

where \( z(t)=Ax(t)+u_0(t) \), \( u_0(t) \) is nominal input.

In this case the, sliding mode dynamic is as follows.

\[ \dot{x}(t) = Ax(t) + Bu_0(t) \]

The sliding mode control law has the following form.

\[ u(t) = u_{eq} + u_{chat}(t) \]

It has a chattering input part. The input chattering can be alleviated by a low pass filtering. By adding the low pass filter, the overall system doesn’t satisfy matching condition at all. Integral sliding mode control is capable of this unmatched system. Now the job is to design the integral sliding mode control including low pass filter. When considering this problem, the first issue is how to determine the nominal control input.

3. MAIN RESULT

The sling mode dynamics have to be same with the original system controlled by nominal controller without uncertainties. As a filter is introduced, the sliding mode has to include filter dynamics. The output of the filter must be the desired nominal control input. The following filter is used.

\[ \dot{u} = -au(t) + u_c(t) \]

The augmented system with filter is expressed as

\[
\begin{bmatrix}
\dot{x}_e(t) \\
\end{bmatrix} =
\begin{bmatrix}
A & B \\
0 & -a \\
\end{bmatrix}
\begin{bmatrix}
x_e(t) \\
u_c(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u_e +
\begin{bmatrix}
h(t) \\
0
\end{bmatrix}
\]

where \( x_e(t) =
\begin{bmatrix}
x(t) \\
u(t)
\end{bmatrix} \)

Integral sliding mode control is used by defining the sliding function as follows.

\[ S = x_e - z_e \]

Where \( z_e =
\begin{bmatrix}
z \\
u_0
\end{bmatrix} \) and \( z =
\begin{bmatrix}
A & B \\
0 & -a \\
\end{bmatrix}
\begin{bmatrix}
x_e(t) + u_{eq}(t) \\
1
\end{bmatrix} \)

\( u_{eq} \) is a nominal input to the augmented system.
Desired control input is the output of the filter. So the input of the filter can make the low pass filter give a desired control input to the controlled system. The nominal input to the augmented system is obtained from the following equation:

\[ u_o(t) = -au_o(t) + u_{eo}(t) \]

In the case of optimal control, the nominal input is

\[ u_o(t) = kx(t) \]

where \( k \) is optimal control gain.

\[ u_{eo}(t) = Kx(t) + au_o(t) \]

\[ = K(Ax(t) + Bu_o(t)) + au_o(t) \]

\[ = KA x(t) + (KB + a)u_o(t) \]

\[ u_o(t) = KB u_o(t) + KA x(t) \]

Integral sliding mode control can be obtained from the following steps.

The Lyapunov candidate function is determined as follows.

\[ v(t) = \frac{1}{2} s^T s \]

The Lyapunov derivative of the sliding function is obtained as follows.

\[ v(t) = S^T (x_e - z_e) \]

\[ = S^T (A \begin{bmatrix} 0 & -a \\ 1 & 0 \end{bmatrix} x_e(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_e(t) + \begin{bmatrix} h(t) \end{bmatrix}) - \]

\[ = S^T (\begin{bmatrix} 0 \\ 1 \end{bmatrix} u_e(t) - u_{eo}(t) + \begin{bmatrix} h(t) \end{bmatrix}) \]

where \( u_e(t) = u_{eo}(t) + u_n(t) \)

\[ = S^T (\begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n(t) + \begin{bmatrix} h(t) \end{bmatrix}) = \begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} h(t) \\ u_n(t) \end{bmatrix} \]

\[ \leq |S_1| |h(t)| + S_2 u_n(t) = \sqrt{S_1^T S_1} \cdot r + S_2 u_n(t) \]

where \( S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = S_1^T h(t) + S_2 u_n(t), |h(t)| < r \)

To make the derivative negative, the following input is used.

\[ u_n(t) = \frac{1}{S_2} \sqrt{S_1^T S_1} \cdot r \]

\[ u_e(t) = u_{eo}(t) - \frac{1}{S_2} \sqrt{S_1^T S_1} \cdot r \]

The resulting value of the Lyapunov derivative is as follows.

\[ \lim_{t \to \infty} S = 0 \]

\[ x_e = \begin{bmatrix} A & B \\ 0 & -a \end{bmatrix} x_e(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{eo}(t) \]

The dynamics of controlled system is as follows.

\[ x(t) = Ax(t) + Bu_o(t) \]

4. NUMERICAL EXAMPLE

The system considered is as follows.

\[ A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad h(t) = \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix}, \quad h_1(t) < r_1 \]

\[ = 2, h_2(t) < r_2 = 3 \]

The filter parameter value is \( a = 2\pi \cdot 5 \)

The sliding surface is following

\[ S = X - Z = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \]

\[ S_1 \]

\[ S_2 \]

\[ S_3 \]

where \( z = \begin{bmatrix} -2 & -3 & 1 \\ 0 & 0 & -a \\ 1 \end{bmatrix} x_e + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{eo} \]

The overall sliding mode controller is following.

\[ u_e = u_{eo} + u_n \]

where \( u_{eo} = (a-1)u + [2 3]x, u_n = -\frac{|S_1|r_1 + |S_2|r_2}{|S_3|} \text{sign}(S_3) \)

The following figures show the computer simulation results.
5. CONCLUSIONS

A low pass filter was added to the original system with uncertainties to alleviated input chattering in SMC. The actual input to the system is the output of the low pass filter. The difficulty of non-matching condition caused by the introducing a low pass filter was solved by introducing the integral sliding mode control. The sliding surface was designed to have the nominal dynamics without uncertainties. The nominal low pass filter input was made to make the filter give the appropriate nominal input to the nominal system.

This work was supported by the Machine Tool Research Center at Changwon National University

REFERENCES


