Attitude Control of Planar Space Robot based on Self-Organizing Data Mining Algorithm

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Abstract: This paper presents a new method for the attitude control of planar space robots. In order to control highly constrained non-linear system such as a 3D space robot, the analytical formulation for the system with complex dynamics and effective control methodology based on the formulation, are not always obtainable. In the proposed method, correspondingly, a non-analytical but effective self-organizing modeling method for controlling a highly constrained system is proposed based on a polynomial data mining algorithm. In order to control the attitude of a planar space robot, it is well known to require inputs characterized by a special pattern in time series with a non-deterministic length. In order to correspond to this type of control paradigm, we adopt the Model Predictive Control (MPC) scheme where the length of the non-deterministic horizon is determined based on implementation cost and control performance. The optimal solution to finding the size of the input pattern is found by a solving two-stage programming problem.

Keywords: Planar Space Robot, Data Mining Algorithm, Model Predictive Control, Caplygin System

1. Introduction

As a new frontier for the human race, space has been attracting great interests. Robotics and its application will be essential to the mission. Among many researches on space robotic systems, the attitude control of artificial satellite system equipped with several manipulators has been recognized as one of the most significant challenges. Space robots are generally restrained by non-holonomic constraints that without external forces and momentum exerted, angular momentum is conserved.

Many approaches have been presented to stabilize the non-holonomic systems, where some of their approaches make use of smooth time-varying or discontinuous control laws[2] [3]. As pointed out by Brockett, however, although the non-holonomic systems are controllable, they are not easily stabilized by applying smooth static state feedback [1]. In our earlier work, a smooth static state feedback control law had been proposed to stabilize the chained system which is one of special classes of non-holonomic system [7]. Another works include [8]. In [8], trajectory tracking control method for non-holonomic systems was proposed, where adaptive control algorithm was applied to consider uncertain parameters.

On the other hand, data mining or data analysis is a new discipline lying at the intersection of statistics and artificial intelligence. Data mining algorithm analyzes observational data sets to find each parameter's relationship with another, and summarizes the data in the way that is useful to the designer. GMDH (Group Method of Data Handling) is a well-known data mining technique that describes suspected dynamics in the form of minimal polynomials.

One of the applications of data mining algorithms is [4], where human driving skills are modeled by polynomial expression. The other applications include [5] and [6] where meteorological model and U.S. interests rate model were constructed. The data mining algorithm is well-defined procedure that takes data as input and produces output in the form of models or patterns. However, its applications are often restricted to conservative problems such as data interpretation, data clustering, pattern recognition etc. These are some types of models constructed by data observation, but attempts to bring out a desired data from the constructed model, are rarely found in literatures. Since the latter is, however, subjected to probabilistic figures, not deterministic analysis, both modeling and control (or data inference) should be considered together. This is because modeling seeks to represent the prominent structures of the data set, and not small idiosyncratic deviations.

This paper presents a new method for attitude control of planar space robots. In order to control highly constrained non-linear systems such as a 3D space robot, the analytical formulation for the system with complex dynamics, and the effective control methodology based on the developed model, are not always obtainable. In the proposed method, correspondingly, a non-analytical but effective self-organizing modeling method for controlling highly constrained systems are proposed based on polynomial data mining algorithm. In order to control the attitude of a planar space robot, it is well-known to require inputs characterized by special pattern in a time series with a non-deterministic length. Once the pattern of inputs is decided, the only parameter that determines control performance is the size of the pattern (which can be translated to the length of the time series). This implies if the input series is determined in a new sampling instant, they cannot be changed before fulfilling all the time series inputs. In order to correspond to this type of control paradigm, we adopt the Model Predictive Control (MPC) scheme where the length of non-deterministic horizon is determined based on implementation cost and control performance. Optimal solution to find the size of the input pattern is found by solving a two-stage programming problem.

2. Dynamics of Planar Space Robot

This chapter presents a brief overview of the system environments. Fig.1 illustrates a planar space robot with two

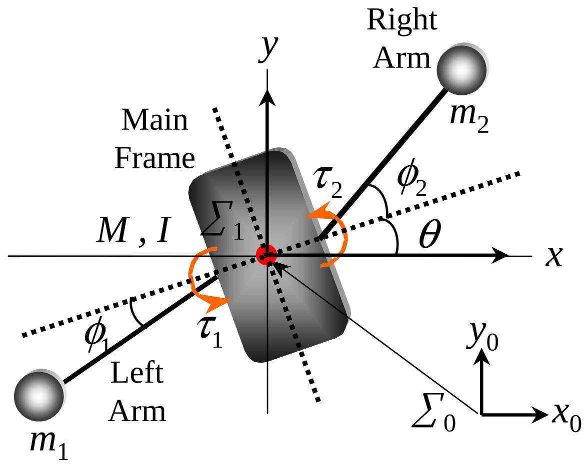

Fig. 1. Rigid Body Model of Planar Space Robot

arms connected to a base satellite via revolute joints, where M and I denote the mass and inertia of the main frame, and m_j and I_j denote the mass and inertia of arm j. The revolute joints are located at a distance r from the base center, and arm link attached to these joints have length l.
At these joints, torque inputs τ_1 and τ_2 actuate the joint angles of the right and left arm. Let θ denotes the attitude of the main frame with respect to $\sum_0$, and ϕ_1 and ϕ_2 denote the angles of the right and left arm with respect to the main frame. Using these parameters, the conservation law of angular momentum and motion equation can be expressed as follows,

$$H(\phi_1, \phi_2) \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} + C(\phi_1, \phi_2, \dot{\phi_1}, \dot{\phi_2}) = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{1}$$

$$\dot{\theta} = \alpha(\phi_1, \phi_2)\dot{\phi_1} + \beta(\phi_1, \phi_2)\dot{\phi_2} \tag{2}$$

, where $H \in R^{2\times 2}$ and $C \in R^2$ are inertia matrix and non-linear forces respectively. Also α and β are smooth functions as follows,

$$\alpha(\phi_1, \phi_2) = \frac{-(ml^2 + mrl\cos\phi_1)}{I + 2mr^2 + 2ml^2 + 2mrl(\cos\phi_1 + \cos\phi_2)} \tag{3}$$

$$\beta(\phi_1, \phi_2) = \frac{-(ml^2 + mrl\cos\phi_2)}{I + 2mr^2 + 2ml^2 + 2mrl(\cos\phi_1 + \cos\phi_2)} \tag{4}$$

This system can be formulated as (5) by introducing inputs $[v_1, v_2]^T$.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ \alpha(x_1,x_2) + \beta(x_1,x_2) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{5}$$

, where x is the state variable vector defined by

$$[x_1, x_2, x_3, x_4, x_5]^T = \left[\theta, \phi_1, \phi_2, \dot{\phi_1}, \dot{\phi_2}\right]^T \tag{6}$$

, and v_1 and v_2 are control inputs that represent angular accelerations of the left and right arms. The system of the equation (5) is a special type of non-holonomic system that is subjected to Phaffian constraint of the following form.

$$J(q)\dot{q} = 0 \tag{7}$$

This system is called *Caplygin system* where Phaffian constraint cannot be integrated to have the algebraic equation form of $H(q) = 0$. The canonical form of Caplygin system can be rewritten by introducing new inputs $[u_1, u_2]$ as follows,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha(x_1,x_2) \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} \beta(x_1,x_2) \\ 0 \\ 1 \end{bmatrix} u_2 \tag{8}$$

, where u_1 and u_2 are angular speeds of left and right arm.

3. Self-Organizing Polynomial Data Mining Algorithm

Since Caplygin system is highly restricted by non-linear Phaffian constraint as shown in the previous section, it is not straightforward to stabilize the system, applying conventional state-variable formulation of modern control theory. It is known for the attitude stabilization of Caplygin system that it requires special type of inputs whose behaviors in a time series are characterized by a pattern on a periodic basis. The problem is how to construct the model that describes the exact behavior of the system. The model should be in a useful form to handle with a control algorithm that for example, aforementioned input pattern (that is considered as useful information to characterize the dynamics of a planar space robot) should be helpfully used. Group Method of Dada Handling (GMDH) was used to extract the relationships between the target variable (to be controlled) and other observational data sets, and compress them in a polynomial expression. The exploratory data analysis by means of GMDH is carried out to model suspected dynamics in a self-organizing manner as follows,

Step 1 Acquisition of Data Sets: The observational data sets are subdivided into a training set and a checking set.

Step 2 Propagation of Variables: All n independent intermediate variables in the training set are combined by partial polynomial as follows.

$$\begin{aligned} z_k = \; & a_{0,k} + a_{1,k}x_{i,p} + a_{2,k}x_{i,q} \\ & + a_{3,k}x_{i,p}^2 + a_{4,k}x_{i,q}^2 + a_{5,k}x_{i,p}x_{i,q} \end{aligned} \tag{9}$$

, where p, q = 1,2, $\cdots$,m, $p \neq q$, k = 1,2, $\cdots$,1/2 $\times$ $m(m+1)$. The coefficients of partial polynomial (9) are obtained by applying the least square method that minimizes the difference between target variable y_i and the dependent variable z_k as follows,

$$J = \sum_{i=1}^{n_{tr}} (y_i - z_k(x_{i,p}, x_{i,q}))^2 \tag{10}$$

, where n_t is the number of data in the training sets.

Step 3 Representativeness Assessment: The adequacy of each newly discovered variables z_k to represent the target variable is measured by following representativeness criterion.

$$r_k = \sqrt{\frac{\sum_{i=n_t+1}^{n}(y_i - z_{k,i})^2}{\sum_{i=n_t+1}^{n} y_i^2}} \quad (11)$$

Here, $z_{k,i}$ denotes i-th data of the variable z_k, where $z_{k,i}$ is obviously in the checking set since $i \geq n_t$.

Step 4 Elimination of Least Effective Variables: The variables $z_{k,i}$ $(1 \leq k \leq n_t)$ are reordered in the order of the size r_k from smallest to biggest. A threshold value R_{th} is introduced and all z_k which do not satisfy $r_k < R_{th}$ are screened out from the data set. The remaining variables are redefined as new x.

Step 5 Model Optimality Test: If $r_{min,l} > r_{min,l+1}$, go to Step 2, otherwise terminate with end, where l is the iteration number, and $r_{min,l}$ is the minimal number of r_k at l-th iteration.

In order to construct the planar space robot model which best associates control objective and observational data available, the intrinsic properties of the system such as an input pattern should be fully considered. once the pattern of the inputs is decided, the only parameter that influences control performance is the size of the input pattern (which can be translated to the length of the time series).
According to the data mining procedure of GMDH, modeling problem for attitude control of a planar space robot can be stated as follows:

> *Find the optimal dynamics that describes the relationship between the size of given input pattern and base angular displacement of planar space robot.*

The following inputs are considered in this paper,

$$\phi_{left} = R(1 - \cos\Gamma(t)) \quad (12)$$

$$\phi_{right} = R(\sin\Gamma(t)) \quad (13)$$

$$\Gamma(t) = \Lambda t - sin(\Lambda t) \quad (14)$$

, where ϕ_{left} and ϕ_{right} denote angular distance of the left and right arm, respectively, and R denotes the variable which determines the size of each input. The input pattern of above equations shows that the left and right arms start to behave in a patterned movement and come back to the start position in a cycle determined by the lengh of $2\pi/\Lambda$. Note that $\Gamma(t)$ is a smooth function with monotonic increase. With this function, the system can be controlled in an effective form that when the system rotates $\theta \gg 0$, the time scale is lengthened, and when the system rotate $\theta \approx 0$, time scale is also shortened. In order to obtain the $R - \Delta_\theta$ relationships in this paper where R is observational data set and angular displacement $\Delta_\theta(\theta(t+\tau) - \theta(t))$ is target variable, GMDH algorithm is applied to the data set. Since the conventional GMDH algorithm, however, needs more than three data sets fully independent with each other to describe target variable, it cannot be applied to our problem at the moment.

Table 1. Coefficients of the polynomial expression developed by GMDH

Coefficient	Value	Coefficient	Value
c_1	-2 × 10^{-7}	c_2	-0.0002
c_3	-2 × 10^{-6}	c_4	0.1256
c_5	3 × 10^{-6}	c_6	-0.021

Table 2. Parameters of Planar Space Robot

Mass of main frame	$M = 3.25\,[Kg]$
Inertia moment of main frame	$I = 6.46 \times 10^{-3}\left[Kgm^2\right]$
Masses of left and right arms	$m_1 = m_2 = 0.868\,[Kg]$
Distance between base center and revolute joint	$r = 0.0500\,[m]$
Length of both arms	$l = 0.450\,[m]$

Therefore, some modifications are carried out in this paper as follows.

Step 4 Elimination of Least Effective Variables: The sets x_p $(1 \leq p \leq m)$ are added to the newly generated variables z_k, where x_p is labeled as z_{n_t+p}. The redefined variable $z_{\bar{k}}$ $(1 \leq \bar{k} \leq n_t + m)$ are reordered in the order of the size $r_{\bar{k}}$ from smallest to biggest. A threshold value R_{th} is introduced to screen out z_k which satisfies $r_k < R_{th}$ from the data set. The remained variables are redefined as new x.

According to this procedure, developed dynamics are shown in the following.

$$\Delta_\theta = \theta(t+\tau) - \theta(t) = c_1 + c_2R + c_3R^2 + c_4R^3 + c_5R^4 + c_6R^5 \quad (15)$$

The coefficients in (15) and parameters used in the numerical experiment are shown in Table 1 and Table 2, respectively. Fig.3 compares the developed $R - \Delta_\theta$ dynamics by means of GMDH and actual angular displacements with respect to R, where the former coincides in all points with the latter.

4. Attitude Control of Planar Space Robot via Model Predictive Control Policy

This section describes an effective method for the attitude control of planar space robot. In order to control the Caplygin system shown in the previous section, structural peculiarities of the system such as input pattern should be fully considered. The input pattern is represented over the time interval that is much longer than sampling interval. This implies if the input series are determined in a new sampling instant, they cannot be changed before fulfilling all the time series inputs. In order to correspond to this type of control paradigm, we adopt the Model Predictive Control (MPC) scheme where the size of non-deterministic horizon is determined based on implementation cost and control perfor-

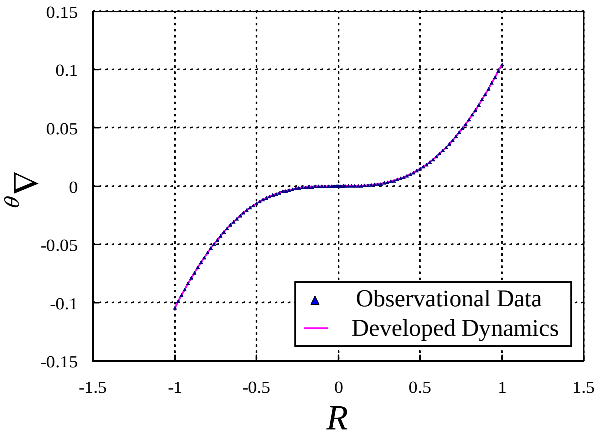

Fig. 2. Developed Dynamics for R-Δ_θ Relationship

mance. The optimal solution to find the size of input pattern that is the only factor to determine control performance, is found by solving a two-stage programming problem.

Based on the model developed in the previous section, the control problem for the attitude control of the planar space robot can be stated as follows:

Find R and Λ which minimize following performance criteria

$$J = (x_3(k_S T_S + T_P(k_P)) - \theta_d)^2 + (T_P(k_P))^2 \tag{16}$$

subject to (3), (4), (8), (12), (13), (14), and (15), where θ_d is the desired value for θ, k_S and k_P are the sampling index and the planning index, and $T_S = t/k_S$ and $T_P = t/k_P$ are the sampling interval and planning interval, respectively.

The first term of (16) imposes the cost as for the difference between x_3 and θ_d and the second term imposes the cost as for the length of the planning interval T_P. Note that the shorter T_P accelerates convergence of x_3 to θ_d. The length of T_P is, however, constrained by (1) that some parameters such as ϕ_1, ϕ_2, $\dot\phi_1$, $\dot\phi_2$, $\ddot\phi_1$, $\ddot\phi_2$ have their maximum values as follows,

$$|\phi_1| \leq \phi_{1,MAX}, \quad |\phi_2| \leq \phi_{2,MAX} \tag{17}$$

$$|\dot\phi_1| \leq \dot\phi_{1,MAX}, \quad |\dot\phi_2| \leq \dot\phi_{2,MAX} \tag{18}$$

$$|\ddot\phi_1| \leq \ddot\phi_{1,MAX}, \quad |\ddot\phi_2| \leq \ddot\phi_{2,MAX} \tag{19}$$

Since x_3 is the function of k_S and $T_P(k_P)$ with a variable length (T_P is also the function of (k_P)), this problem is in the class of a non-linear programming problem that is burdened with large computational efforts. This paper, correspondingly, proposes a two-stage programming based two-stage method as follows,

Step 1 Find R which minimizes

$$\begin{aligned} J &= (x_3(k_S T_S + T_P) - \theta_d)^2 \\ &= (x_3(k_S T_S) + \Delta_\theta - \theta_d)^2 \\ &= (x_3(k_S T_S) + a_1 + a_2 R + a_3 R^2 \\ &\quad + a_4 R^3 + a_5 R^4 + a_6 R^5 - \theta_d)^2 \end{aligned} \tag{20}$$

subject to (17) and

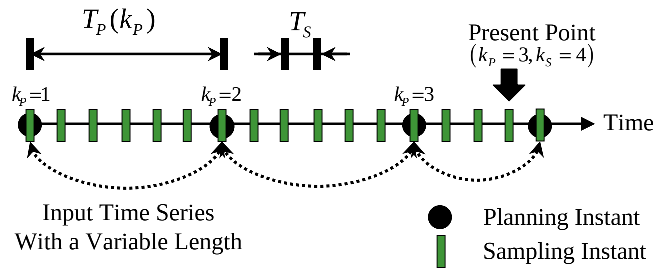

Fig. 3. Proposed Model Predictive Control Policy

$$R_{MIN} \leq |R| \leq R_{MAX} \tag{21}$$

, where R_{MAX} and R_{MIN} are physically constrained maximal and minimal value of R, and (17) can be represented using R as follows,

$$R \leq \frac{\phi_{1,MAX}}{1 - \max\{\cos(\Gamma(k_S))\}} \tag{22}$$

$$R \leq \frac{\phi_{2,MAX}}{\max\{\sin(\Gamma(k_S))\}}. \tag{23}$$

Step 2 Find the minimal input horizon $T_P(k_P) = \frac{2\pi}{\Lambda}$ which satisfies (18), (19) and

$$\Lambda_{MIN} \leq \Lambda \tag{24}$$

, where Λ_{MIN} is the physically constrained minimal value of Λ, and (18) and (19) can be represented using Λ as follows,

$$\Lambda \leq \arg_\Lambda\{\dot\phi_{1,MAX}\} \quad \Lambda \geq \arg_\Lambda\{\dot\phi_{1,MIN}\} \tag{25}$$

$$\Lambda \leq \arg_\Lambda\{\dot\phi_{2,MAX}\} \quad \Lambda \geq \arg_\Lambda\{\dot\phi_{2,MIN}\} \tag{26}$$

$$\Lambda \leq \arg_\Lambda\{\ddot\phi_{1,MAX}\} \quad \Lambda \geq \arg_\Lambda\{\ddot\phi_{1,MIN}\} \tag{27}$$

$$\Lambda \leq \arg_\Lambda\{\ddot\phi_{2,MAX}\} \quad \Lambda \geq \arg_\Lambda\{\ddot\phi_{2,MIN}\}. \tag{28}$$

Note that each parameter of the above equations is easily found that for example,

$$\arg_\Lambda\{\ddot\phi_{2,MAX}\} = \sqrt{\frac{\ddot\phi_{2,MAX}}{4R}}.$$

Fig. 4 shows the proposed model predictive control policy with the variable input horizon $T_P(k_P)$. At every planning instant $k_P(k_S = 1)$ in Fig. 4, the controller firstly finds R that is a type of measure for the size of input pattern, and then obtains the input horizon $T_P(k_P)$. At the next sampling instant $k_S = 2$, the controller put out corresponding values of inputs as follows,

$$u_1(k_S) = R(1 - \cos(\Gamma(k_S))) \tag{29}$$

$$u_2(k_S) = R(\sin(\Gamma(k_S))) \tag{30}$$

, where

$$\Gamma(k_S) = \Lambda k_S T_S - \sin(\Lambda k_S T_s). \tag{31}$$

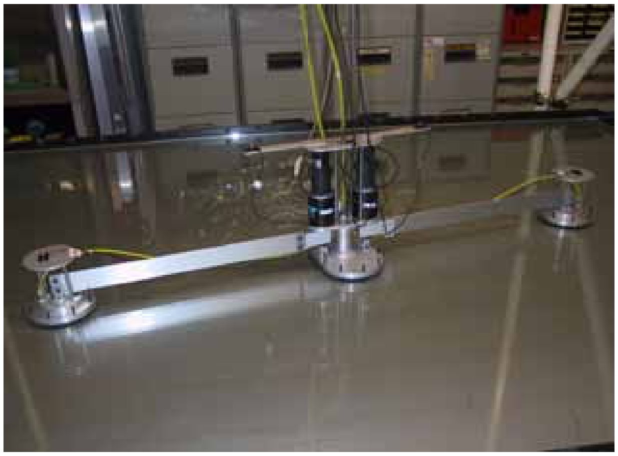

Fig. 4. Planar Space Robot

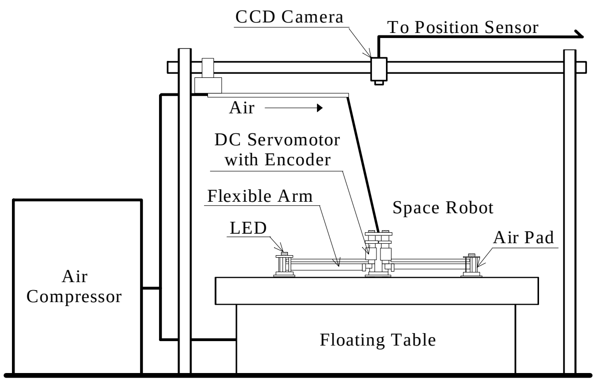

Fig. 5. Outline of Experimental Apparatus

5. Experimental Results

In this subsection, the experimental results are provided to investigate a usefulness of the proposed modeling and control method for the planar space robot. The prototype system is shown in Fig.4. Fig.5 shows the outline of experimental apparatus. In Fig.5, the planar space robot is on the flat table, where this table is covered with glass. Each parameter of the system is shown in TABLE II. Note air pads installed at the center of the main frame and at the end of the two arms. Through them, compressed air is exhausted on the table in order to minimize friction between the robot system and the ground. Maximum power of the air compressor is 3.7[kW]. The floating table with the size 2400 × 1500 [mm] is perfectly leveled on the ground.

Two DC servomotors equipped on the main frame of the robot actuate two arms separately. The angles of the arms with respect to the main frame (ϕ_1 and ϕ_2 of Fig.2) are measured with encoders. Maximum power of the DC servomotors is 18.5[W]. CCD camera mounted on the ceiling detects signals of LEDs installed at the end of the arms and measures movement of the robot. Sensory information taken by the camera is not used for controlling but only for modeling the R-Δ_θ dynamics of the system.

Fig.6 illustrates the data transfer unit of experimental apparatus. Joint angle data measured by encoders and position data taken by CCD camera are transferred to DSP through

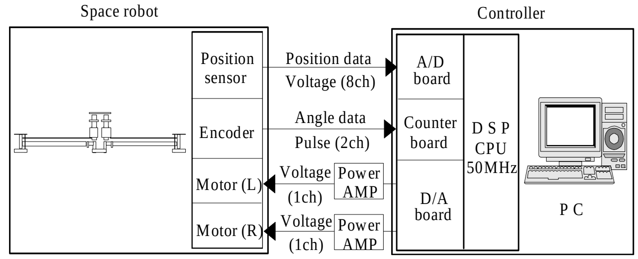

Fig. 6. Data Transfer Unit of Experimental Apparatus

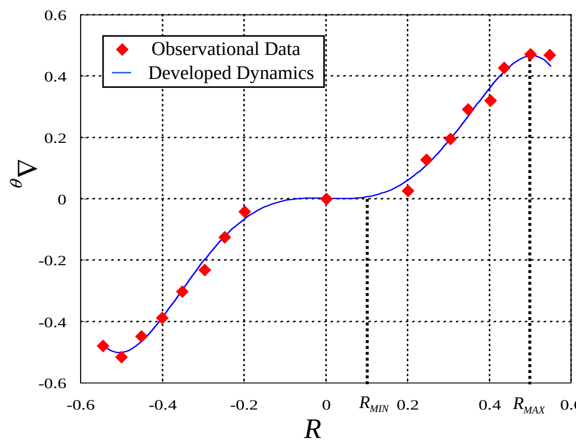

Fig. 7. Behaviors of $R - \Delta_\theta$ Relationship

A/D converter and counter board. Then control signals computed by DSP are sent to the amplifiers of DC servomotors through the D/A board.

We firstly show the result obtained by applying the proposed modeling method to real system. In Fig.7, the points marked with diamonds indicate the sampled data obtained by applying the inputs whose size of pattern is varied from -0.55 to 0.55. The $R - \Delta_\theta$ dynamics was obtained as follows,

$$\begin{aligned}\Delta_\theta &= -8 \times 101^{-6}R^2 + 0.124712R^3 \\ &\quad +0.000021R^4 - 0.020329R^5 - 0.000013R^6\end{aligned} \tag{32}$$

In Fig.7 the trajectory within $-R_{MIN} \leq R \leq R_{MIN}$ plots almost 0 value, and the trajectory at $R = R_{MAX}$ plots the inflected point that we only use this dynamics in the regions of $R_{MIN} \leq R \leq R_{MAX}$ and $-R_{MAX} \leq R \leq -R_{MIN}$

Fig.9 and Fig.9 plots planned value of inputs u_1 and u_2, and their actual values. Although the planned values of inputs are smooth curves, actual implementation to the system contains some vibration. This is because the arms of the prototype system are made with aluminum beams that with some power exerted, they vibrate. The trajectory of x_3, however, well converges to 0 [rad] in Fig.8. We see that the effectiveness of the proposed method is confirmed.

6. Conclusions

This paper has presented a new method for attitude control of planar space robots based on a self-organizing polynomial data mining algorithm. We firstly proposed a new modeling method, taking advantage of the structural peculiarities of

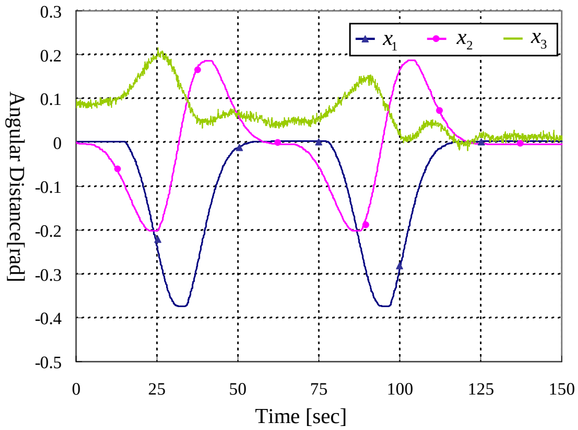

Fig. 8. Behaviors of x_1, x_2 and x_3

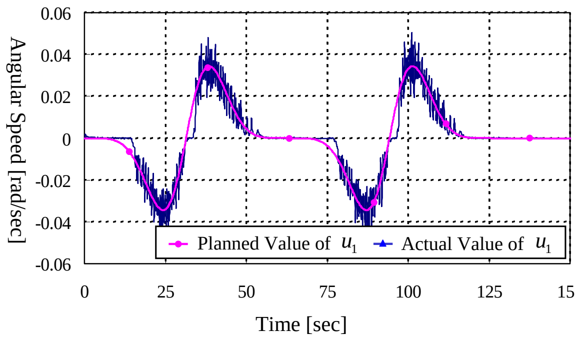

Fig. 9. Behaviors of u_1

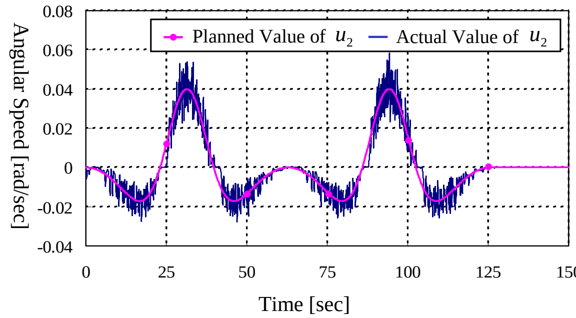

Fig. 10. Behaviors of u_2

the system such as input pattern. The developed model is used for stabilizing the system where two-stage method is applied to obtain optimal size of the given inputs. The proposed method has been formulized based on model predictive control policy with variable length of input horizon. Lastly, the usefulness of the proposed method has been confirmed through some experiments.

Acknowledgements

This work was supported by High-Tech Research Center, Project for Private Universities: matching fund subsidy from MEXT (Ministry of Education, Culture, Sports, Science and Technology), 2002-2006.

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