Hybrid PD - Servo State Feedback Control Algorithm for Swing up Inverted Pendulum System

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Abstract: In this paper, a hybrid PD-servo state feedback control algorithm for swing up inverted pendulum system is proposed. It consists of two parts. The first part is the PD position control for swinging up the pendulum from the natural pendent position to around the upright position and the second part is the servo state feedback control for stabilizing the inverted pendulum in upright position. The first controller is PD controller and it is tuned to control the position of the pendulum by moving the cart back and forth until the pendulum swings up around the upright position. Then the second controller will be switched to stabilize the inverted pendulum in its upright position. The controller in this stage is the servo state feedback controller designed by pole placement. Experimental results of PD type swinging up control system, of stabilizing servo state feedback control system and of the proposed hybrid PD-servo state feedback control system to swing up and stabilize inverted pendulum show that the proposed method is effective and reliable for actual implementation while it is simple.

Keywords: Inverted Pendulum, Swinging-up Control, PD Position Control, Pole-placement Method

1. INTRODUCTION

As a practical plant, swing up inverted pendulum is an old but challenging problem in the field of nonlinear control study. An inverted pendulum, consisting of a cart and pendulum system, has a structure where the pendulum is hinged to the cart via a pivot and only the cart is actuated. Swing up inverted pendulum has many advantages in theoretical study such as simple structure, nonlinear and uncertain characteristic. There are many control methods are tested in swing up inverted pendulum. For example, swing up control using Qubit neural network [1], fuzzy control algorithm [2], robust swing up control [3], nonlinear control [4] and energy-based methods [5]. Though good performances can be obtained by using these algorithms, they are very complicated to implement and their parameters are not easy to design.

In this paper, a controller designed by using hybrid PD-servo state feedback control algorithm to swing up the pendulum from the natural pendent position to around the upright position and to stabilize inverted pendulum in upright position is proposed. It composes of two parts. The first part is the PD position control and the second part is the servo state feedback control. In the first part, the controller is a PD controller for swinging up the pendulum from the natural pendent position to around the upright position. The PD controller is tuned by using trial and error method to obtain the appropriate gains resulting in fast step response with small overshoot. The PD controller controls the motion of the pendulum by moving the cart back and forth within a limited travel of the cart. The cart will be controlled to move to the opposite direction when the angular velocity of the pendulum at the stop point of each travelling is zero. The cart movement will be repeated to next assigned cart position until the inverted pendulum swung up around the upright position. When this condition has been reached, the second part, which is a servo state feedback controller, will be switched to stabilize the inverted pendulum in its upright position. In this paper, this servo state feedback controller is designed by pole placement method.

The proposed control scheme is implemented to control the inverted pendulum on cart as shown in Fig. 3. The effectiveness of the proposed method and the reliability of actual implementation have been shown by the experimental results.

2. MATHEMATICAL MODEL

The inverted pendulum on cart to be controlled is shown in Fig.1. Its structure composes of a cart and pendulum where the pendulum is hinged to the cart via a pivot and only the cart is actuated. θ is the pendulum angle (rad), x is the cart position (m), M is the mass of the cart (kg), m is the mass of the pendulum (kg), l is the distances from turning center to center of mass of the pendulum (m), f is the cart's friction coefficient (kg/s) and F is the force applied to the cart (N).

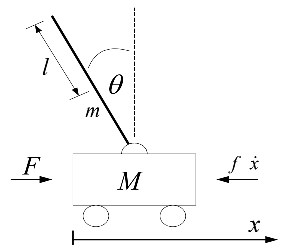


Fig. 1 Inverted pendulum on cart.

Lagrange's equations are applied with respect to θ and x coordinates and the state variables are consequently assigned as $x_1 = \theta, x_2 = \dot{\theta}, x_3 = x$ and $x_4 = \dot{x}$ where the input u is the applied force F. The nonlinear state space model of inverted pendulum on cart system can then be obtained as

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{\left(mg\sin x_1 + \dfrac{m\cos x_1\left(u - f\,\dot{x}\right) - m^2 l x_2^{\,2}\cos x_1 \sin x_1}{\left(M+m\right)} \right)}{\left(\dfrac{4}{3}ml - \dfrac{m^2 l\cos^2 x_1}{\left(M+m\right)} \right)} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{\left(u - f\,x_1 + \dfrac{3}{4}mg\cos x_1 \sin x_1 - mlx_2^{\,2}\sin x_1 \right)}{\left(\left(M+m\right) - \dfrac{3}{4}m\cos^2 x_1 \right)}
\end{aligned} \tag{1}
$$

The inverted pendulum system has two distinct equilibrium points concerning the pendulum angle. The first equilibrium point is at the upright position which is unstable and the second equilibrium point is at the hanging position.

In order to design an appropriate controller, the system is linearized about the upright position of the inverted pendulum. The following linear state equation for the inverted pendulum on cart can be obtained as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \tag{2}$$

where

$$
\mathbf{A} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
\dfrac{3\left(M+m\right)mg}{4ml\left(M+m\right) - 3m^2 l} & 0 & 0 & \dfrac{-3mf}{4ml\left(M+m\right) - 3m^2 l} \\
0 & 0 & 0 & 1 \\
\dfrac{3mg}{4\left(\left(M+m\right) - \dfrac{3}{4}m\right)} & 0 & 0 & \dfrac{-f}{\left(M+m\right) - \dfrac{3}{4}m}
\end{bmatrix},
$$

$$
\mathbf{B} = \begin{bmatrix}
0 \\
\dfrac{3m}{4ml\left(M+m\right) - 3m^2 l} \\
0 \\
\dfrac{1}{\left(M+m\right) - \dfrac{3}{4}m}
\end{bmatrix},
$$

and where $\mathbf{x}^T = \left[x_1\ x_2\ x_3\ x_4\right]^T = \left[\theta\ \dot{\theta}\ x\ \dot{x}\right]^T$. In this case, the input u or the applied force F is generated by DC motor to move the cart of the inverted pendulum system.

As the main interest is to control the position x of the cart of the inverted pendulum system; therefore, the following output equation

$$y(t) = \mathbf{C}\mathbf{x}(t) \tag{3}$$

can be given, where $\mathbf{C} = \left[0\ 0\ 1\ 0\right]$.

3. CONTROL SYSTEM STRUCTURE

The overall structure of the proposed system is shown in Fig. 2. The first controller is a PD controller for controlling the inverted pendulum to its upright position and the second controller is a servo state feedback controller for stabilizing the inverted pendulum when it is in its upright position. The details of each controller will be described in the following sub-sections.

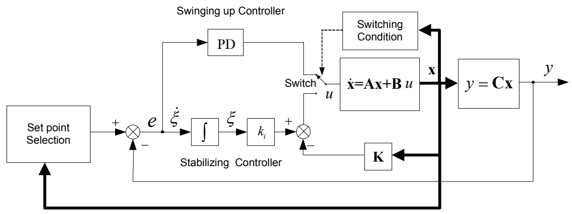


Fig. 2 Structure of the proposed control system.

3.1 PD position control

The objective task for the PD position control shown in Fig. 2 is to swing up the inverted pendulum from the natural pendent position to around the upright position before switching to the stabilizing control mode. In this paper, the transfer function of the PD controller is given by

$$u(t) = k_p e(t) + k_d \frac{d}{dt}e(t) \tag{4}$$

where k_p is the proportional gain, k_d is the derivative gain and e is the cart position error. The PD controller will control the cart to move back and forth consecutively depending on pre-assigned cart position of which the pendulum staying at the natural pendent position is excited. The back and forth movement of the cart will be occurred when the angular velocity of the pendulum at the present position is zero. The PD controller in this paper is proposed to tune to achieve the fast response without overshoot. These movements proceed until the pendulum can be swung up around the upright position where the second controller will be switched to stabilize the pendulum.

3.2 Servo state feedback control

The objective task of the servo state feedback control shown in Fig. 2 is to stabilize the inverted pendulum around the upright position. From Fig. 2, it is seen that

$$\dot{\xi} = r - y = r - \mathbf{C}\mathbf{x}\,. \tag{5}$$

Since an integral is added to tact the output of the system without steady-state error; therefore, the following augmented system for designing a servo state feedback controller can be obtained as

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{-C} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{x}(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}u(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}r(t) \tag{6}$$

$$y(t) = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{x}(t) \\ \xi(t) \end{bmatrix} \tag{7}$$

and its control law will be given by

$$u(t) = -\begin{bmatrix} \mathbf{K} & -k_i \end{bmatrix}\begin{bmatrix} \mathbf{x}(t) \\ \xi(t) \end{bmatrix} \tag{8}$$

where $\mathbf{K}$ is the state feedback gain matrix, k_i is the integral gain, ξ is the output of the integral controller and r is the reference signal. The gains $\mathbf{K}$ and k_i can either be assigned by linear quadratic regulator (LQR) approach or pole-placement method.

4. EXPERIMENTAL RESULTS

In this section, the proposed hybrid PD-servo state feedback control algorithm for swinging up and stabilizing the inverted pendulum on cart will be implemented. The experimental apparatus is shown in Fig. 3. The parameter values of the inverted pendulum system for obtaining its mathematical model are shown in Table 1.

Fig. 3 Experimental apparatus

Table 1 Parameter values of inverted pendulum

M (kg)	m (kg)	l (m)	f (kg/s)
0.642	0.123	0.25	0.165

The linear model of the system must be known for assigning the proposed controllers. By utilizing the values shown in Table 1, the linear model of the inverted pendulum on cart in eq. (2) can be obtained, where

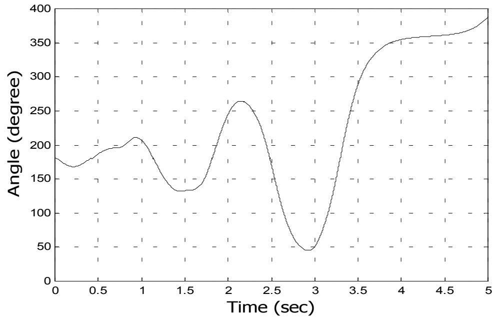
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 24.77 & 0 & 0 & -7.35 \\ 0 & 0 & 0 & 1 \\ 1.34 & 0 & 0 & -0.19 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ 4.45 \\ 0 \\ 1.48 \end{bmatrix}.$$

It can be noticed here that the model of the inverted pendulum on cart has the open-loop poles at s = 0, 4.77, -5.17, 0.20 of which the system is unstable.

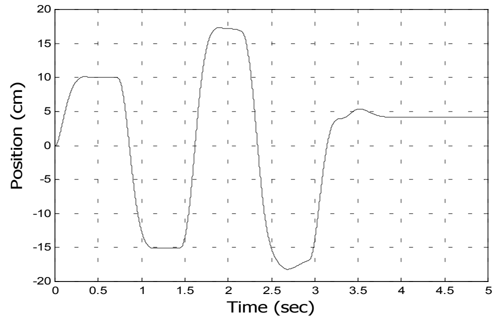
4.1 Swing up responses

The objective task of the PD position control is to bring the inverted pendulum up around the upright position as fast as possible. By using the trial and error method, the desired proportional gain k_p and derivative gain k_d of PD type swinging up controller are selected as 30 and 5 respectively. The sequential travelling positions of cart are assigned to be $10, -15,\ 17, -18$ and 5 cm respectively. The pendulum can be swung up around the upright position using zero angular velocity condition occurred at the stop point of the present position to move the cart to the next assigned position.

The experimental results obtained from a real machine are shown in Fig. 4. It is shown in Fig. 4 (a) that the PD controller can swing pendulum up to around the upright position (360 degrees) within 4.5 seconds while the moving response of the cart corresponding to the pre-assigned positions is shown in the Fig. 4 (b).



(a) Angle of pendulum



(b) Cart position

Fig. 4 Experimental results of swinging up control.

4.2 Stabilizing responses

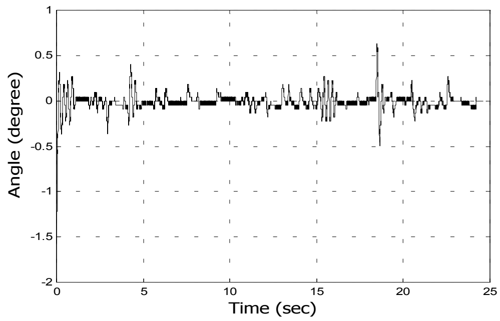
The gains $\mathbf{K}$ and k_i of the servo state feedback control which has a main task of stabilizing the inverted pendulum are assigned by using pole-placement method. In this experiment, it is desired that the percent overshoot is 15 % and the settling time is 5 seconds. Hence, the corresponding closed-loop poles of the inverted pendulum system are at $s = -1 \pm j\sqrt{3}, -5, -5$ and -5. This yields $\mathbf{K} = [70.46 \quad 15.27 \quad -17.83 \quad -34.50]$ and $k_i = -16.21$. The initial values of the cart position x and the pendulum angle θ are zeroes.

Applying the servo state feedback controller to the actual system, the stabilizing responses are obtained. Figure 5 shows responses of the cart position and the angle of pendulum. The experimental result shows that the controller has good performance in controlling the cart position and pendulum angle around the zero-degree lines.

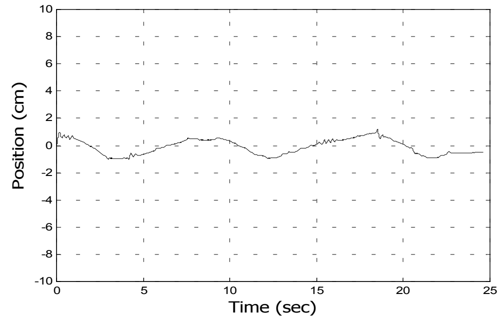
4.3 Swing up and stabilizing responses

In this sub-section, the simultaneous control of the inverted pendulum from swinging up control to stabilizing control will be investigated. The stabilizing control will be switched when the pendulum is around the upright position. The switching condition is selected as $|\theta| < 0.25$ rad and $|\dot{\theta}| < 0.1$ rad/s.

The experimental results of proposed control system are shown in Fig. 6. It can be seen from Fig. 6 (a) that the pendulum can be swung up from the natural pendent position to upright position around 5 seconds by PD controller and then the system is switched to stabilizing control mode to stabilize the pendulum in its upright position. It can also be seen that the cart position response oscillates around the zero-degree line.

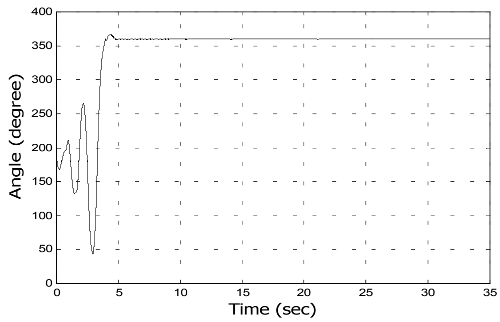


(a) Angle of pendulum

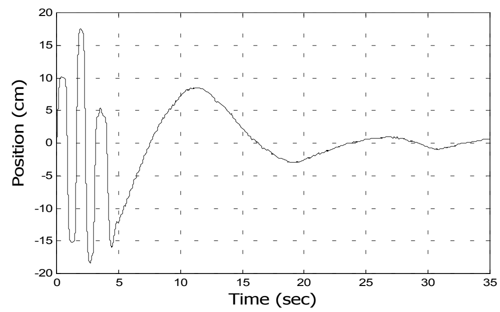


(b) Cart position

Fig. 5 Experimental results of stabilizing control.



(a) Angle of pendulum



(b) Cart position

Fig. 6 Experimental results of swinging up and stabilizing controls of inverted pendulum.

5.CONCLUSION

The hybrid PD-servo state feedback control algorithm consisting of the PD position control for swinging up the pendulum from the natural pendent position to around the upright position and the servo state feedback control for stabilizing the inverted pendulum in its upright position has been proposed in this paper. According to the cooperative tasks of PD controller and servo state feedback controller, the control of inverted pendulum on cart has been realized. In conclusion, the proposed method is effective and yields the desired system performance despite its simplicity in design.

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