Experimental Examination of Multivariable PID Controller Design on Frequency Domain using Liquid Level Process

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Abstract: This paper is concerned with the examination and evaluation concerning a tuning method of multivariable PID controllers based on partial model matching on frequency domain proposed by authors from practical view point. In this case, PID controller parameters are determined by minimizing the loss function defined by the difference between frequency response of ideal model transfer function and actual frequency response on several frequency points. The purpose of the paper is to examine and evaluate the performance of the method through actual experiments of MIMO liquid level experimental process control equipment.

Keywords: Process control, PID control, Multi-inputs/multi-outputs system, Partial model matching on frequency domain, Liquid level control, Frequency response

1. Introduction

PID control has occupied the position of the most utilizing technique in process control through over sixty years because of its simplicity and robustness. In SISO case, we already have had several well-known PID controller parameter tuning methods which include, for example, Ziegler-Nichols method and so on [1]. On the contrary, we have a few methods on the tuning of MIMO PID controllers through most systems are considered as MIMO systems actually[2], [3], [4].

Recently, authors have proposed a method to tune MIMO PID controllers based on partial model matching on frequency domain. The special feature of the method is that it requires only several frequency response of the plant at several frequency points to tune PID controller parameters. That is, it is not necessary to know exact numerical model of the actual plant to tune PID controllers. Thus it can easily be applied to MIMO case. This method was first developed by authors as a tool of SISO PID controller parameter tuning[5]. The method was modified and applied to MIMO PID controller parameters tuning[6], [7], [8]. The effectiveness of the method was confirmed by applying it to several mechanical systems. For example, we have used it to the design of active vibration control system and obtained a good result. However, as to the application of the method to the process control, examination of its effectiveness was made only through numerical simulation. One of the difficulties concerning practical application of the method arises from the difficulty of obtaining the frequency response or spectrum of the process. To avoid such a problem, in this report, we proposed to use step response of the plant because step response is generally easy to obtain in process system. Here we use the so-called Prony's method to obtain transfer function from the step response of the plant. As a result we can easily obtain frequency response at any desired frequency points of the plant. The effectiveness of the proposed design procedure is examined and evaluated through an experimental tests using 2-inputs/2-outputs experimental liquid level equipment.

This paper is organized as follows. In Section 2, we derive the control sytem design problem to be treated here briefly. The MIMO PID parameter tuning method based on partial model matching on frequency domain is explained in Section 3. Evaluation of the method using 2-inputs/2-outputs liquid level process is shown in Section 4 with several experimental results.

2. Problem setup

Let us consider the following m-inputs/m-outputs plant shown in Fig.1.

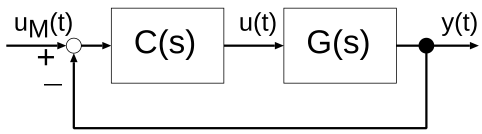


Fig. 1. Block diagram of control system.

In Fig.1, $m \times m$ transfer function matrices $\boldsymbol{G}(s)$ and $\boldsymbol{C}(s)$ denote the plant transfer function matrix and PID controller matrix, respectively. Their elements are defined as follows:

$$\boldsymbol{G}(s) = [g_{ij}(s)]_{i,j=1,\cdots,m} \ , \tag{1}$$

$$\begin{aligned} \boldsymbol{C}(s) &= [c_{ij}(s)]_{i,j=1,\cdots,m} \\ &= \left[k_{P_{ij}} + \frac{k_{I_{ij}}}{s} + \frac{k_{D_{ij}} s}{1+\gamma_i s} \right] . \end{aligned} \tag{2}$$

$\boldsymbol{u}_M(t) = [u_{M_i}(t)]_{i=1,\cdots,m}$ is a reference input vector, $\boldsymbol{u}(t) = [u_i(t)]_{i=1,\cdots,m}$ is a control input vector and $\boldsymbol{y}(t) = [y_i(t)]_{i=1,\cdots,m}$ is an output vector of the system, respectively. Note that the parameter γ_i in Eq.(2) is introduced for practical realizability of the differential term.

From Fig.1 we can obtain the open-loop transfer function matrix of the control system as follows:

$$\begin{aligned} \boldsymbol{Q}(s) &= \boldsymbol{G}(s)\boldsymbol{C}(s) = [q_{ij}(s)]_{i,j=1,\cdots,m} \\ &= \left[\sum_{p=1}^{m} g_{ip}(s) c_{pj}(s) \right]_{i,j=1,\cdots,m} . \end{aligned} \tag{3}$$

Further we consider the following reference control system shown in Fig.2 where $\boldsymbol{G}_M(s)$ is the desired open-loop transfer function matrix.

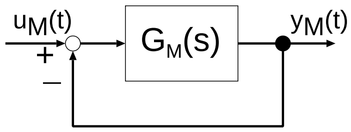


Fig. 2. Block diagram of Reference system

For the simplicity, we give $\boldsymbol{G}_M(s)$ in the following diagonal form.

$$\boldsymbol{G}_M(s) = \mathrm{diag}[g_{M_i}(s)]_{i=1,\cdots,m} \ . \tag{4}$$

The desired closed-loop transfer function matrix is given as follows:

$$\begin{aligned} \boldsymbol{G}_{M_{cl}}(s) &= (\boldsymbol{I}_m + \boldsymbol{G}_M(s))^{-1} \boldsymbol{G}_M(s) \\ &= \mathrm{diag}\left[\frac{g_{M_i}(s)}{1+g_{M_i}(s)}\right]_{i=1,\cdots,m} \ . \end{aligned} \tag{5}$$

Obviously, it is impossible to realize exact model matching between $\boldsymbol{Q}(s)$ and $\boldsymbol{G}_{M_{cl}}(s)$ because of the limitation concerning the PID controller matrix $\boldsymbol{C}(s)$.

3. Parameter tuning policy of MIMO PID controller

In this section, we will show the tuning policy of MIMO PID controller parameters based on partial model matching method with stability constraints[7], [8].

3.1. Loss function

Consider the model matching between $\boldsymbol{G}_M(j\omega)$ and $\boldsymbol{Q}(j\omega)$ on some frequency set

$$\Omega = [\boldsymbol{\Omega}_1, \cdots, \boldsymbol{\Omega}_m] \ . \tag{6}$$

where $\boldsymbol{\Omega}_i$ is defined as follows:

$$\boldsymbol{\Omega}_i = \left[\omega_{i_1}, \cdots, \omega_{i_{N_i}}\right], \ i = 1, \cdots, m \ . \tag{7}$$

To evaluate the difference of frequency responce between reference model and control system on $\boldsymbol{\Omega}_i$, we introduce the following error functions:

$$\epsilon_{ii}(j\omega_{i_k}) = \frac{g_{M_i}(j\omega_{i_k}) - \sum_{p=1}^{m} g_{ip}(j\omega_{i_k}) c_{pi}(j\omega_{i_k})}{g_{M_i}(j\omega_{i_k})} \ , \tag{8}$$

$$\begin{aligned} \epsilon_{qi}(j\omega_{i_k}) &= \frac{\sum_{p=1}^{m} g_{qp}(j\omega_{i_k}) c_{pi}(j\omega_{i_k})}{g_{M_q}(j\omega_{i_k})}, \\ & \omega_{i_k} \in \boldsymbol{\Omega}_i, \quad i, q = 1, \cdots, m, \quad q \neq i \ . \end{aligned} \tag{9}$$

Then we can define the loss function as follows:

$$\begin{aligned} J_i(\boldsymbol{\theta}_i) &= \sum_{k=1}^{N_i} |\epsilon_{ii}(j\omega_{i_k})|^2 + \sum_{k=1}^{N_i} |\epsilon_{ii}(-j\omega_{i_k})|^2 \\ &+ \sum_{\substack{q=1 \\ q\neq i}}^{m} \left(\sum_{k=1}^{N_i} |\epsilon_{qi}(j\omega_{i_k})|^2 + \sum_{k=1}^{N_i} |\epsilon_{qi}(-j\omega_{i_k})|^2 \right), \\ & \omega_{i_k} \in \boldsymbol{\Omega}_i, \quad i = 1, \cdots, m \ . \end{aligned} \tag{10}$$

The PID parameter vector $\boldsymbol{\theta}_i$

$$\boldsymbol{\theta}_i = [k_{P_{i1}}, k_{I_{i1}}, k_{D_{i1}}, \cdots, k_{P_{im}}, k_{I_{im}}, k_{D_{im}}]^T_{i=1,\cdots,m} \tag{11}$$

is included in Eq.(10). Then the problem is to find an optimal PID parameters $\boldsymbol{\theta}_i$, $i = 1, \cdots, m$, which minimize the loss function Eq.(10). In Eq.(10), $J_i(\boldsymbol{\theta}_i)$ is derived as a sum of square and its conjugate form. The mean of adding the complex conjugate term is to obtain $\boldsymbol{\theta}_i$ as a real parameter value when we apply the least squares to the minimization problem of Eq.(10) [9].

3.2. Stability conditions

As stated in 3.1, the PID parameters $\boldsymbol{\theta}_i$ which minimize loss function on $\boldsymbol{\Omega}_i$ can be obtained as the least squares solution of Eq.(10). However, thus obtained solution does not guarantee the stability of the actual closed-loop system. To improve the situation, a numerical method of solving Eq.(10) with some stability constraints are proposed[8] based on Rosenblock's stability theorem[10].
To realize the above stated objective, we consider the following 2 sufficient conditions for the stability of the system:

Condition 1

Nyquist plot of diagonal elements $q_{ii}(s), i = 1, \cdots, m$ of $\boldsymbol{Q}(s)$ always looks at $(-1, j0)$ on its left hand side on $\omega \in [0, \infty)$.

Condition 2

The i-th row or column Gershgorin bands does not include $(-1, j0)$ for $\omega \in [0, \infty)$.

Obviously, we cannot guarantee these two conditions in the design of our PID controller design scheme because we have to treat the design problem on the finite frequency point set $\boldsymbol{\Omega}_i$. However, it might be useful to consider the attainability of these conditions from practical sense though it is imperfect from theoretical point of view. To this end, we consider the following procedures.

(1) Suppose that the Nyquist plot of $g_{M_i}(j\omega)$ looks at $(-1, j\omega)$ on $\omega \in [0, \infty)$ and diagonal constraint frequency $\omega_{M_{cl_i}}$ be phase cross-over frequency of $g_{M_i}(j\omega)$. Further suppose that $q_{ii}(j\omega_{M_{c_i}})$ locates inside the small region of the circle (center; $g_M(j\omega_{M_{c_i}})$, radius; $r_i(j\omega_{M_{c_i}})$). Then, it is expected that the Nyquist plot of $q_{ii}(j\omega)$ takes a position to look at $(-1, j0)$ to the left hand side for all $\omega \in [0, \infty)$.

This condition can be written on $\boldsymbol{\Omega}_i$ as follows:

$$\begin{aligned} & \left| g_{M_i}(j\omega_{M_{c_i}}) - \sum_{p=1}^{m} g_{ip}(j\omega_{M_{c_i}}) c_{pi}(j\omega_{M_{c_i}}) \right|^2 \\ & + \left| g_{M_i}(-j\omega_{M_{c_i}}) - \sum_{p=1}^{m} g_{ip}(-j\omega_{M_{c_i}}) c_{pi}(-j\omega_{M_{c_i}}) \right|^2 \\ & \leq \left| r_i(j\omega_{M_{c_i}}) \right|^2 + \left| r_i(-j\omega_{M_{c_i}}) \right|^2, \qquad i = 1, \cdots, m \end{aligned} \tag{12}$$

where radius $r_i(j\omega_{M_{c_i}})$ is given by

$$r_i(j\omega_{M_{c_i}}) = \delta_i \left| g_{M_i}(j\omega_{M_{c_i}}) \right|, \quad \delta_i > 0 \tag{13}$$

$$\delta_i \left| g_{M_i}(j\omega_{M_{c_i}}) \right| < \left| 1 + g_{M_i}(j\omega_{M_{c_i}}) \right| \ . \tag{14}$$

(2) If the circle with radius $\sum_{\substack{r=1\\r\neq i}}^{m} |q_{ri}(j\omega_{ni_k})|$ centered at $|1+q_{ii}(j\omega_{ni_k})|$ does not include $(-1, j0)$ on non-diagonal constraint frequency set $\boldsymbol{\Omega}_{nd_i} = [\omega_{n i_1}, \cdots, \omega_{n i_P}]$, $0 < \omega_{ni_k} < \infty$, $i = 1, \cdots, m$, then the condition 2 is satisfied at least on $\boldsymbol{\Omega}_{nd_i}$.

Of course, this condition is insufficient from the theoretical view point. However, it is practically sufficient if we can choose sufficiently large number of $\boldsymbol{\Omega}_{nd_i}$. The above statement is formulated as follows:

$$\begin{aligned}
&\left| 1 + \sum_{p=1}^{m} g_{ip}(j\omega_{ni_k}) c_{pi}(j\omega_{ni_k}) \right|^2 \\
&+ \left| 1 + \sum_{p=1}^{m} g_{ip}(-j\omega_{ni_k}) c_{pi}(-j\omega_{ni_k}) \right|^2 \\
&\geq \alpha_i \left\{ \left| \sum_{\substack{q=1\\q\neq i}}^{m} \left| \sum_{p=1}^{m} g_{qp}(j\omega_{ni_k}) c_{pi}(j\omega_{ni_k}) \right| \right|^2 \right. \\
&\left. + \left| \sum_{\substack{q=1\\q\neq i}}^{m} \left| \sum_{p=1}^{m} g_{qp}(-j\omega_{ni_k}) c_{pi}(-j\omega_{ni_k}) \right| \right|^2 \right\} \\
&\alpha_i > 1, i = 1, \cdots, m, \omega_{ni_k} \in \boldsymbol{\Omega}_{nd_i}
\end{aligned} \tag{15}$$

In the following, we summarize the MIMO PID parameter tuning algorithm mentioned above.

Step 1. Estimate the frequency response of $\boldsymbol{G}(s)$ on $\boldsymbol{\Omega}_i, i = 1, \cdots, m$.

Step 2. Give the diagonal reference model $\boldsymbol{G}_M(s)$.

Step 3. Select diagonal constraint frequencies $\omega_{M_{cl_i}}$ and non-diagonal constraint frequency set $\boldsymbol{\Omega}_{nd_i}$ $(i = 1, \cdots, m)$.

Step 4. Give the design parameters δ_i, α_i and γ_i $(i = 1, \cdots, m)$.

Step 5. Solve optimization problem under stability constraints formulated in the following:

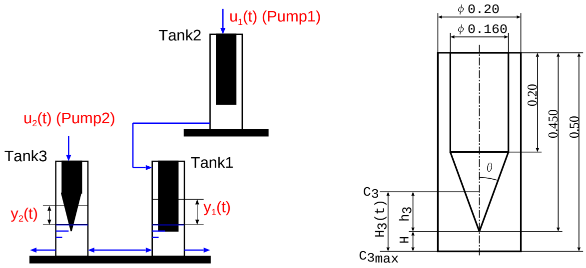
"minimize$J_i(\boldsymbol{\theta}_i)$ with respect to $\boldsymbol{\theta}_i$
subject to constraints Eq.(12) and Eq.(15) ."

Note: Step 5 is attainded by solving a non-linear programming problem defined above. Here we used a MATLAB Tool Box[11].

4. Experiment

4.1. Experimental equipment

We examine and evaluate the effectiveness of the MIMO PID parameter tuging algorithm derived in the preceding section by using the liquid level process shown in Fig.3. In Fig.3, $y_1(t)$ and $y_2(t)$ are liquid level increments from steady states of Tank1 and Tank3, respectively. $u_1(t)$ and $u_2(t)$ describe the control inputs of the system. Each tank includes solid cylinder and/or cone. Note that the section of Tank3: C_3



Liquid level process Tank3

Fig. 3. Liquid level process

varies with respect to the change of $y_2(t)$. That is, Tank3 has the following nonlineality:

$$C_3 = C_{3\max} - \pi h_3^2 \tan^2(\theta) [\mathrm{m}^2] \ . \tag{16}$$

where $h_3 = H_3(t) - H$, $0 < h_3 \leq 0.05$[m], $\theta = 0.309$[rad/sec]. This nonlinearity is neglected in the transfer function approximation.

4.2. Controller design

(1) Process identification (determination of $\boldsymbol{G}(j\omega_k)$)

It is necessary to determine $\boldsymbol{G}(j\omega_k)$, not $\boldsymbol{G}(j\omega)$, at finite frequency points ω_k for applying the above stated method. Unfortunately, different from the mechanical system's case, it is rather difficult to obtain frequency response data in process systems. Instead, we utilize the result of step response data. Here we used the so-called Prony's method to obtain the transfer function matrix of the process from the step response data. This procedure was recently developed by authors[12], [13]. As a fact, we obtained the following transfer functions:

$$g_{11}(s) = \frac{13.3s + 0.6915}{s^2 + 0.01654s + 3.148 \times 10^{-5}} \tag{17}$$

$$g_{12}(s) = \frac{34.9953s + 0.5240}{s^2 + 0.0179s + 3.5313 \times 10^{-5}} \tag{18}$$

$$g_{21}(s) = \frac{0.195}{s^2 + 0.02913s + 3.109 \times 10^{-5}} \tag{19}$$

$$g_{22}(s) = \frac{35.24s + 0.5343}{s^2 + 0.01813s + 3.621 \times 10^{-5}} . \tag{20}$$

Bode diagrams of Eqs.(17) - (20) are shown in Fig.4.

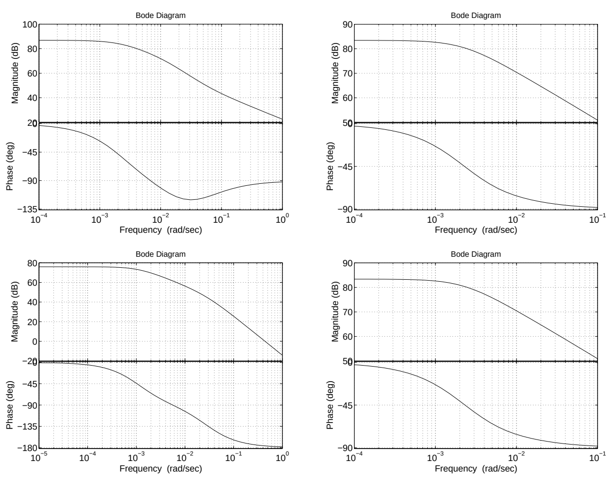


Fig. 4. Bode diagrams of $\boldsymbol{G}(j\omega)$

(2) Selection of reference model

First, we select diagonal reference model $g_{M_i}(s)$. From the preceding facts, Nyquist plot of $g_{Mi}(s)$ must have enough gain and phase margin because we have to take into the sufficient stable margin from the view point of Gershgorin circle. Here we choose a closed-loop reference model such as

$$\boldsymbol{G}_{M_{cl}}(s) = \mathrm{diag}\left[\frac{\omega_{MO_i}^3}{s^3 + \zeta_i \omega_{M0_i} s^2 + \zeta_i \omega_{M0_i}^2 s + \omega_{M0_i}^3}\right]_{i=1,2} .$$

It corresponds to the binomial coefficient model when $\zeta_i = 2$ and its cross-over frequency is: $\omega_{M_{c_i}} = \sqrt{\zeta_i}\omega_{M0_i}$. It is noted that the corresponding open-loop reference model is

$$\boldsymbol{G}_M(s) = \mathrm{diag}\left[\frac{\omega_{M0_i}^3}{s^3 + \zeta_i \omega_{M0_i} s^2 + \zeta_i \omega_{M0_i}^2 s}\right]_{i=1,2} .$$

In this example, we set

$$\begin{aligned} \omega_{M0_1} &= 0.0983 \text{ [rad/sec]} , \\ \omega_{M0_2} &= 0.0856 \text{ [rad/sec]} . \end{aligned} \tag{21}$$

So that the reference model is given as follows:

$$\boldsymbol{G}_M(s) = \begin{bmatrix} \frac{0.000512}{s^3+0.24s^2+0.0192s} & 0 \\ 0 & \frac{0.000343}{s^3+0.21s^2+0.0147s} \end{bmatrix} , \tag{22}$$

and the cross-over frequencies of the reference model become

$$\begin{aligned} \omega_{M_{c_1}} &= 0.139 \text{ [rad/sec]}, \\ \omega_{M_{c_2}} &= 0.121 \text{ [rad/sec]} . \end{aligned} \tag{23}$$

In this case, gain margin and phase margin of $g_{M_i}(s)$ become 20[dB] and 1.4[rad/sece], respectively. Taking into consideration that the gain margin is about $3 \sim 10$[dB] and phase margin is 0.35[rad/sec] in usual case, the selected referense model is considerably stable.

(3)Comments on the selection of matching frequencies

(3.1) $\boldsymbol{\Omega}_i$.

According to the reference[9], the number of matching frequency N_i must satisfy the inequality

$$N_i \geq \frac{3m}{2} . \tag{24}$$

In this example, we took the minimum number: $N_i = 3$. Interval of 3 frequency points are selected equivalently in logarithmic scale. It is also recommended to choose these 3 points are smaller than the value of $\omega_{M_{c_i}}$[7]. Here we select

$$\boldsymbol{\Omega}_i = \left[1.0 \times 10^{-4}, 3.16 \times 10^{-4}, 1.0 \times 10^{-3}\right]_{i=1,2} \text{ [rad/sec]} . \tag{25}$$

(3.2) Diagonal constraint frequency and phase cross-over frequency.

$$\boldsymbol{\Omega}_{d_1} = 0.139, \quad \boldsymbol{\Omega}_{d_2} = 0.121 \text{ [rad/sec]} . \tag{26}$$

$$\boldsymbol{\Omega}_{nd_1} = 0.139, \quad \boldsymbol{\Omega}_{nd_2} = 0.121 \text{ [rad/sec]} . \tag{27}$$

(3.3) Other weighting parameters.

$$\delta_i = 1.0, \quad \alpha_i = 6.0, \quad \gamma_i = 0.1, \quad i = 1, \cdots, m . \tag{28}$$

(4) Optimal PID controller parameters (Result of computation)

Result of computation, 4 PID controller parameters $\boldsymbol{\theta}_i$ are given as follows:

$$\begin{aligned} \boldsymbol{k}_{11} &= [6.14 \times 10^{-4},\ 0.0166 \times 10^{-4},\ -1.69 \times 10^{-3}] \\ \boldsymbol{k}_{12} &= [-3.89 \times 10^{-4},\ -0.0147 \times 10^{-4},\ -1.83 \times 10^{-2}] \\ \boldsymbol{k}_{21} &= [0.230 \times 10^{-4},\ -0.00670 \times 10^{-4},\ -2.37 \times 10^{-3}] \\ \boldsymbol{k}_{22} &= [4.76 \times 10^{-4},\ 0.0217 \times 10^{-4},\ -2.71 \times 10^{-3}] \end{aligned} \tag{29}$$

where $\boldsymbol{k}_{ij} = [k_{P_{ij}}, k_{I_{ij}}, k_{D_{ij}}],\ i, j = 1, 2$.

4.3. Result

(1) Matching and Stability

To confirm the satisfaction of partial model matching, frequency response of $\boldsymbol{Q}(j\omega)$ is calculated and compared with frequency response of $\boldsymbol{G}_M(j\omega)$. The results are shown in Fig.5 and Fig.6 by Bode diagrams. In these figures, solid line shows $\boldsymbol{Q}(j\omega)$, dashed line shows $\boldsymbol{G}_M(j\omega)$ and circle shows frequency response at matching frequencies. Though it is not necessary that calculating frequency responses which correspond to matching frequencies, entire frequency response are shown on Fig.5. To confirm the satisfaction of conditions 1 and 2, we give Fig.7 and Fig.8. These graphs show the satisfaction of conditions 1 and 2.

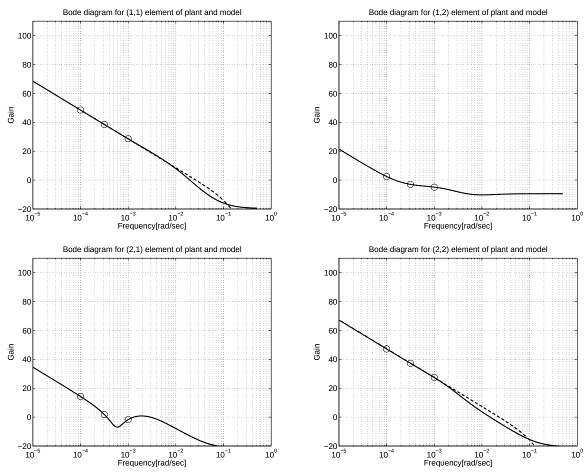


Fig. 5. Matching of Loop Transfer Function between $\boldsymbol{G}(s)\boldsymbol{C}(s)$ and $\boldsymbol{G}_M(s)$ (Gain)

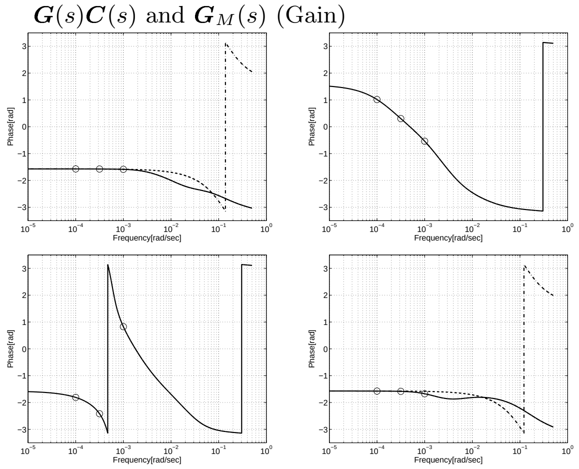


Fig. 6. Matching of Loop Transfer Function between $\boldsymbol{G}(s)\boldsymbol{C}(s)$ and $\boldsymbol{G}_M(s)$ (Phase)

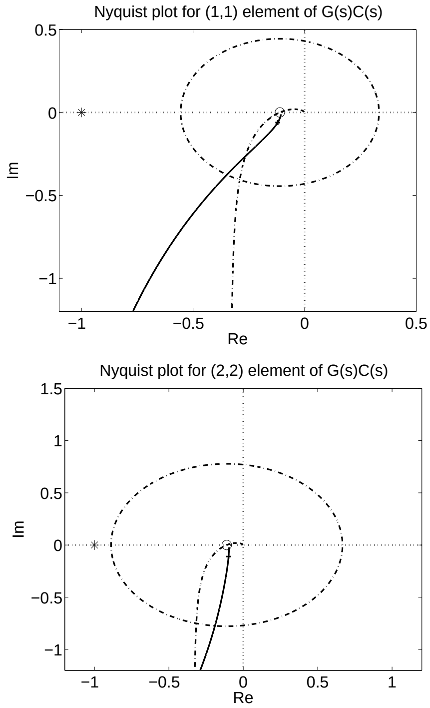


Fig. 7. Nyquist Plots of Diagonal Elements of $\boldsymbol{G}(s)\boldsymbol{C}(s)$

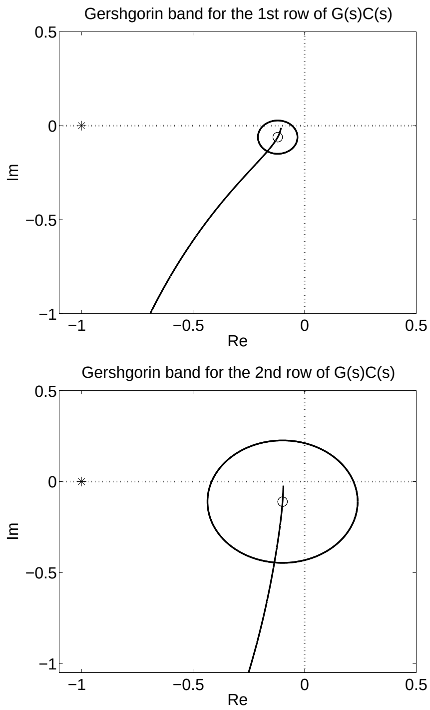


Fig. 8. Gershgorin Disks for 1st and 2nd Rows of $\boldsymbol{G}(s)\boldsymbol{C}(s)$

From these results, we can conclude that the control system was designed to achieve model matching on wide frequency range although the number of matching frequencies is only a few points.

(2) Step responses

Case1-1

In Case1-1, reference input $\boldsymbol{u}_M(t)$ was set as follows:

$$u_{M_1}(t) = 0.01, \quad u_{M_2}(t) = 0.0 \text{ [m]} . \tag{30}$$

Case1-2

In Case1-2, reference input $\boldsymbol{u}_M(t)$ was set as follows:

$$u_{M_1}(t) = 0.0, \quad u_{M_2}(t) = 0.01 \text{ [m]} . \tag{31}$$

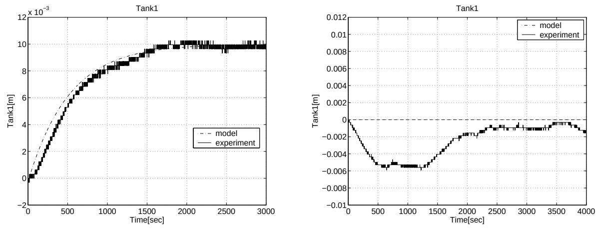


Fig. 9. Outputs of the plant and the reference model (Case1-1)

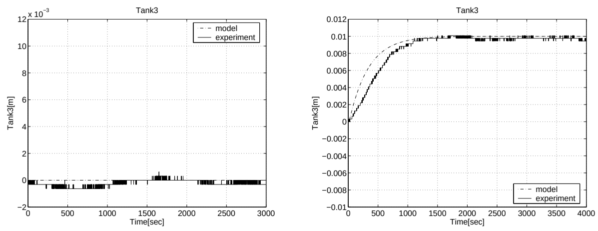


Fig. 10. Outputs of the plant and the reference model (Case1-2)

Fig.9 and Fig.10 show the result of Case1-1 and Case1-2, respectively. In these figures, solid line shows the plant output, dash line shows the reference model output.

From these results, we can conclude that decoupling and tracking performance are both well attained in the experiments. It is noted that the step inputs are small enough so that the linearity of the plant are kept in both cases.

To examine the tolerance concerning nonlinearlity, we executed the following experiments Case2-1, Case2-2. As being shown in Fig.3, the section of Tank3 varies with $y_2(t)$. Table1 shows actual section changes corresponding to measured $y_2(t)$.

Table 1. Nonlinearity of Tank3's section area $C_3[\text{m}^2]$ with respect to $y_3(t)$[m]

	Case.1-2		Case.2-2	
	$y_2(t)$	C_3	$y_2(t)$	C_3
Initial	0.194	0.0248	0.186	0.0255
steady state	0.204	0.0239	0.226	0.0215

Note that from Table.1, the rate of section change of Tank3 is 15.6% in Case1-2, though it is 3.6% in Case2-2. This facts mean that the rate of time constant change of Tank3 approximated to first-order lag system is 15.6% in Case1-2, though it is 3.6% in Case2-2.

Case2-1

In Case2-1, reference input $\boldsymbol{u}_M(t)$ was selected as follows:

$$u_{M_1}(t) = 0.04, \quad u_{M_2}(t) = 0.0 \text{ [m]} \tag{32}$$

Case2-2

In Case2-2, reference input $\boldsymbol{u}_M(t)$ was selected as follows:

$$u_{M_1}(t) = 0.0, \quad u_{M_2}(t) = 0.04 \text{ [m]} \tag{33}$$

Fig.11 and Fig.12 show the results of Case2-1 and Case2-2.

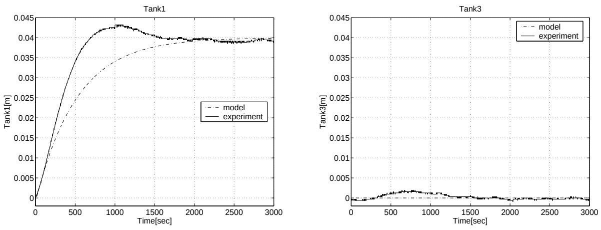


Fig. 11. Outputs of the plant and the reference model (Case2-1)

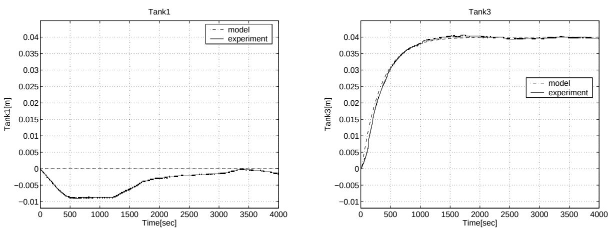


Fig. 12. Outputs of the plant and the reference model (Case2-2)

Compared to the results of Case1, the obtained result shown in Fig.11 and Fig.12 are not so good as to the tracking performance. However, taking into account that the step height of reference output becomes 4 times larger than Case1-1 and Case1-2, we can say that the control performanse is still kept well for the increase of nonlinearlity.

5. Conclusion

In this paper, we applied MIMO PID controller tuning method based on partial model matching on frequency domain to liquid level process control. The effectiveness of the proposed tuning method to process control is examined through experiments. It must be emphasized that the model matching between control system and reference system on frequency domain was well achieved in spite that the number of matching frequency is only 3. Moreover, we could confirm that the controller designed by the method was effective against nonlinearity in some extent in our experimental experience.

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