Nonlinear Adaptive Control Law for ALFLEX
Using Dynamic Inversion and Disturbance Accommodation Control Observer

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Abstract: In this paper, we present a new nonlinear adaptive control law using a disturbance accommodating control (DAC) observer for a Japanese automatic landing flight experiment vehicle called ALFLEX. A future spaceplane must have ability to deal with greater fluctuations in the stability and control derivatives of flight dynamics, because its flight region is much wider than that of conventional aircraft. In our previous studies, digital adaptive flight control systems have been developed based on a linear-parameter-varying (LPV) model depending on dynamic pressure, and obtained good simulation results. However, under previous control laws, it is difficult to accommodate uncertainties represented by disturbance and nonlinearity, and to design a stable flight control system. Therefore, in this study, we attempted to design a nonlinear adaptive control law using the DAC Observer and inverse dynamic methods. A good tracking property of the obtained system was confirmed in numerical simulation.

Keywords: Guidance and Control, Adaptive Control, Disturbance Accommodating Control Observer, Spaceplane, ALFLEX

1. INTRODUCTION

In this paper, a new nonlinear adaptive control law using a disturbance accommodation control observer is presented for a Japanese automatic landing flight experiment vehicle called ALFLEX. ALFLEX’s flight experiments have been carried out to demonstrate the overall system technology and evaluate automatic landing technology for future Japanese spaceplanes. In this study, we focus on the guidance and control law for ALFLEX. In particular, the purpose of this paper is to make the roll rate and yaw rate of the vehicle track their references in designing the guidance and control law.

In general, a spaceplane encounters greater fluctuations in the stability and control derivatives of flight dynamics because its flight region is much wider than that of conventional aircraft. Therefore, a conventional linear-time-invariant (LTI) expression is not appropriate in dealing with such a spaceplane’s characteristics, so that the employment of some sort of adaptive mechanism is required.

Recently, a theory called the “linear adaptive theory” has been proposed by Johnson, of the University of Alabama, USA. Using this theory with a disturbance accommodating control (DAC) observer [6,7], a nonlinear controlled system can be linearized by constructing feedback loops consisting of the terms estimated by the DAC observer and can also be provided with adaptability, because, the DAC observer estimates the disturbance, uncertain and coupling terms of the controlled system. Thus, a MIMO nonlinear system can be transformed into a decoupled linear system, and the asymptotic stability of the system is guaranteed using the subspace stabilization technique. This stabilization technique is for finding feedback gains such that the state variables of the obtained closed-loop system asymptotically approach a subspace that satisfies prescribed boundedness and stability conditions.

Therefore, in this study, we attempt to apply the linear adaptive theory and subspace stabilization control technique with the DAC observer to a first-time-scale subsystem obtained using inverse dynamics transformation and singular perturbation approaches.

Finally, to examine the effectiveness of the proposed system, numerical simulation was performed with ALFLEX’s 6-degree-of-freedom nonlinear simulation model.
2. CONTROL LAW FOR ALFLEX

2.1 ALFLEX Dynamics

On the assumption of the flat-Earth model, a 6-degree-of-freedom of ALFLEX’s dynamics can be given by the following equations [8].

(a) Translational equations of motion

\[ \ddot{V} = (T \cos \alpha \cos \beta - D + m \lambda g) / m \]  
\[ \dot{\alpha} = (-L - T \sin \alpha + m B g) / m V \cos \beta \] 
\[ \dot{\beta} = (Y - T \cos \alpha \cos \beta + m C g) / m V + p \sin \alpha - r \cos \alpha \]

Here,
\[ A = -\cos \alpha \cos \beta \sin \theta + \sin \beta \sin \phi \cos \theta \] 
\[ B = \cos \alpha \cos \phi \cos \theta + \sin \alpha \sin \theta \] 
\[ C = \cos \alpha \sin \beta \sin \theta + \cos \beta \sin \phi \cos \theta \] 
\[ -\sin \beta \cos \phi \cos \theta \]

(b) Rotational equations of motion

\[ \dot{\epsilon} p = \frac{1}{I_{xx} l_{zz} - I_{xz} l_{xy}} \left\{ I_{xx} l_{zz} - I_{xz} l_{xy} + I_{zz} l_{xx} - I_{xy} l_{xz} \right\} \]  
\[ \dot{\epsilon} q = \frac{1}{I_{yy}} \left\{ M - I_{zz} \left( p^2 - r^2 \right) - (I_{yy} - I_{zz}) q r \right\} \]  
\[ \dot{\epsilon} r = \frac{1}{I_{zz} l_{xx} - I_{xx} l_{yy}} \left\{ I_{yy} l_{xx} - I_{xx} l_{yy} + I_{xx} l_{zz} - I_{yy} l_{xy} \right\} \]

(c) Relationship between rates of Euler angles and body-axis angles

\[ \dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta \]  
\[ \dot{\theta} = p \cos \phi - r \sin \phi \]  
\[ \dot{\psi} = (q \sin \phi + r \cos \phi) \sec \theta \]

2.2 Dynamic Inversion and Singular Perturbation

The inverse dynamics transformation theory is a technique that sequentially cancels a plant’s characteristic by sequentially forming an inversion system (dynamics), and realizes an ideal transient response determined arbitrarily by a system designer. On the other hand, the singular perturbation theory is an approach in which a control system is separated into fast-time-scale (FTS), middle-time-scale (MTS) and slow-time-scale (STS) subsystems according to the difference in response time between the state variables, and each feedback loop is constructed according to each time scale. Hence, for a slower subsystem, the state variables of a faster subsystem can be assumed to become steady values. Thus, in the proposed method, the order of the system can be reduced by employing time scale separation, so that numerous state variables can be controlled.
In eq.(18), we consider $[\bar{v} \; u]^T$ to be inputs such that eq.(18) can be written as

$$\dot{x} = K_f(\bar{v} - x) + K_z \int_0^t (\bar{v} - x) dt.$$  \hfill (20)

From eqs.(18) and (20), $[\bar{v} \; u]^T$ is generated using

$$[\bar{v} \; u] = [B_f(\bar{v}) \; C_f(\bar{v})]^T [-A_f(\bar{v}) + (\bar{v} - x) + K_z \int_0^t (\bar{v} - x) dt].$$  \hfill (21)

Substituting $\bar{v}$ from eq.(21) into eq.(19), $\bar{w}$ can be obtained. However, since this $\bar{w}$ is obtained by assuming $\bar{v} \to 0$, there exists a difference between the actual variable $z$ and $\bar{w}$ and the FTS variable $\bar{v}$ and $\bar{w}$. Therefore, to decrease both differences, we consider the next system called FTS.

Therefore, to maintain $\Delta z$ close to zero.

$$\Delta z = A_f(z - \bar{v}), \quad \Delta w = w - \bar{w}.$$  \hfill (23)

In this system, the FTS control input $\Delta w$ can be designed for Eq.(22) to maintain $\Delta z$ close to zero.

### 3. LINEAR ADAPTIVE CONTROL LAW

#### 3.1 Linearization of system using DAC observer

As mentioned above, ALFLEX’s dynamics is treated as a nonlinear system including disturbance. As controlled variables, we consider roll rate and yaw rate in the FTS subsystem. Therefore, eqs. (7) and (9) are expressed by the following nonlinear differential equations.

$$\dot{p} = m_s(p, r, w) + L_{\delta_\alpha} \delta_e$$  \hfill (24)

$$\dot{r} = m_s(p, r, w) + N_{\delta_p} \delta_r$$  \hfill (25)

Here, the first term represents the coupling, uncertain and disturbance terms and the second term represents the control input term. Since eqs.(24) and (25) have similar forms, henceforth we will consider to design an observer for the first equation eq.(24).

First, we define the error in roll rate as

$$e_p = p^* - p.$$  \hfill (26)

Differentiating this, the differential equation with respect to roll rate error is obtained.

$$\dot{e}_p = p^* - \dot{p} = p^* - m_s(p, r, w) - L_{\delta_\alpha} \delta_e$$  \hfill (27)

Here, $p^*$ represents the predefined reference roll rate. Denoting the first and second uncertain terms as $z_{p*} = p^* - m_s(p, r, w)$, eq. (27) can be rewritten as

$$\dot{e}_p = z_{p*} - L_{\delta_\alpha} \delta_e.$$  \hfill (28)

Here, to estimate the unknown term $z_{p*}$, we employ the DAC observer. To do so, we consider a quadratic spline function with respect to time.

$$\dot{z}_{p*} = f(t) = c_1 + c_2 t + c_3 t^2$$  \hfill (29)

Thus, eqs.(29) and (30) can be rewritten into the following forms according to reference [6].

$$\dot{e}_p = z_{p*} - L_{\delta_\alpha} \delta_e$$  \hfill (30a)

$$\dot{z}_{p*} = I \dot{z}_{p*}$$  \hfill (30b)

Here,

$$z_{p*} = [z_{p1}; z_{p2}; z_{p3}]^T, I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$  \hfill (30c)

As an observer for eqs.(31a) and (31b), we introduce

$$\hat{z}_{p*} = z_{p*} - L_{\delta_\alpha} \delta_e$$  \hfill (31a)

$$\dot{\hat{z}}_{p*} = I \dot{\hat{z}}_{p*}$$  \hfill (31b)

Subtracting eqs.(32a) and (32b) from eqs.(31a) and (31b), respectively, and combining the results, the following DAC observer is obtained.

$$\begin{bmatrix} \hat{e}_p - \dot{\hat{z}}_{p*} \\ \dot{\hat{z}}_{p*} - \hat{z}_{p*} \end{bmatrix} = \begin{bmatrix} -k_{11} & 1 & 0 \\ -k_{21} & 0 & 0 \end{bmatrix} \begin{bmatrix} e_p - \hat{e}_p \\ \hat{z}_{p*} - \hat{z}_{p*} \end{bmatrix}$$  \hfill (33)

If observer gain $k = [k_1 \; k_2 \; k_3]$ is selected so that the above equation becomes stable, $[\hat{e}_p \; \hat{z}_{p*}]$ approaches $[e_p \; \hat{z}_{p*}]$ with time.

Next, to design the tracking system for the roll rate, as a state variable, we consider the integral of tracking error, tracking error and its derivative.

$$x = [x_1 \; x_2 \; x_3]^T = \int_0^t e_p \; \hat{e}_p \; \dot{\hat{e}}_p \; \dd t.$$  \hfill (34)

As an actuator of the vehicle, we assume the following first-order lag element with the time constant $T_s$.

$$\delta_e = -\frac{1}{T_s} \delta_e + \frac{1}{T_s} u_p.$$  \hfill (35)

Differentiating eq.(34) and using eq.(35), the state differential equation becomes

$$\begin{bmatrix} \frac{d}{dt} x_1 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{1}{T_s} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u_p.$$  \hfill (36)

To simplify the problem, we divide the total input into two parts.

$$u_p = u_{p, h} + u_{p, r}.$$  \hfill (37)

Here, $u_{p, h}$ is a control so that the second term in eq.(36) is cancelled out.

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\[ u_{p,h} = \frac{T}{L_{\delta_0}} \left( \frac{\dot{\theta}_m}{T} + \dot{\dot{\theta}}_p \right) \] (38)

where, \( u_{p,h} \) is a control that guarantees the asymptotic stability of the system.

Substituting eqs. (37) and (38) into eq. (36), a linearized equation is obtained with regard to rolling rate error.

\[
\dot{x} = Ax + Bu
\]

Here, the purpose of the control is to make the state variable \( x \) promptly follow a prescribed value, \( x^* \), which represents an acceptable error response in accordance with an ideal model dynamics.

\[
\dot{e}_p + \alpha_1 e_p + \alpha_2 \int_0^t e \, d\tau = 0
\] (40)

Here, the feedback gains \( \alpha_1 \) and \( \alpha_2 \) are determined according to the subspace stabilization technique described in the next section.

### 3.2 Subspace-Stabilization Control Technique

First, the subspace-stabilization control technique is briefly summarized here according to references [6,7].

The subspace-stabilization control problem is to find the feedback control \( u = K(t)x \) such that all solutions \( x(t) \) of the close-loop system \( \dot{x} = A(t)x + B(t)u \) asymptotically approach \( S \) as \( t \to \infty \) and such that all the motions \( x(t) \in S \) satisfy prescribed boundedness or stability conditions. The surface \( S \) represents the desired time behavior of the error state components through

\[
S : \begin{bmatrix} \alpha_0 & \alpha_1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0
\] (41)

At this point, we consider a certain coordinate transformation from the \( x \) space to the \( \xi \) space.

\[
x = \begin{bmatrix} C \xi \\ M \xi \end{bmatrix}. \] (42)

The inverse transform from \( \xi \) to \( x \) is

\[
\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} C & M \end{bmatrix} x,
\] (43)

where the norm of the subvector \( \xi_1 \in \mathbb{R}^I \) constitutes a measure of the distance to the subspace \( S \) and the subvector \( \xi_2 \in \mathbb{R}^L \) lies on the subspace \( S \).

From the above definition of the transformations, the following relation must be satisfied.

\[
\begin{bmatrix} C & \xi_2 \end{bmatrix} \begin{bmatrix} C \xi_1 \\ M \xi_1 \end{bmatrix} = I, \quad CM = 0
\] (44)

As matrices that satisfy such a condition, we can select

\[
C^L = C (CC^T)^{-1} \quad \text{and} \quad M^L = (M^L M)^{-1} M^T.
\] (45)

Here, \( M = \begin{bmatrix} m_1 & m_2 \end{bmatrix} \) consists of two linearly independent columns and \( C \) is any \((3 - 2) \times 3\) matrix such that \( CM = 0 \).

In the new \( \xi \) space, differentiating eq.(43) and using the feedback \( u = K(t)x \), the original state equation

\[
\dot{x} = A(t)x + B(t)u
\]

is transformed to

\[
\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} C & \xi_2 \end{bmatrix} \begin{bmatrix} A + BK[C - C^L] \\ M + [A + BK][M - M^L] \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix},
\]

(46)

Here, for system (46), we assume that the following two conditions are satisfied.

\[
(A(t) + B(t)K(t))M(t) = M(t)R(t), \quad \text{for some } 2 \times 2 \text{ square matrix } R(t)
\]

\[
C(t)(A(t) + B(t)K(t)) + C(t) = V(t)C(t), \quad \text{for some scalar } V(t)
\]

Thus, eq.(46) can be simplified to

\[
\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} V & 0 \\ M^L [A + BK][C - C^L] \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}.
\]

(49)

For the system state \( x(t) \) to promptly approach \( S \), that is, \( \xi_1 \to 0 \), within a specified settling time, the reduced system

\[
\dot{\xi}_1 = V \xi_1
\]

must be asymptotically stable to the equilibrium point \( \xi_1 = 0 \).

Therefore, employing \( V = -\mu, \mu > 0 \), and substituting it into eq.(49), the feedback gain that satisfies eq.(49) becomes

\[
K = \frac{\mu \alpha_1 T}{L_{\delta_0}} \begin{bmatrix} \mu \alpha_1 \dot{x}_1 + \alpha_2 x_2 + \left( \mu + \alpha_1 - \frac{1}{T} \right) x_3 \end{bmatrix}
\]

and the control input for achieving stability becomes

\[
u_{p,h} = \frac{T}{L_{\delta_0}} \begin{bmatrix} \dot{x}_p + \dot{\dot{\theta}}_p + \mu \alpha_2 \dot{x}_1 \\ \mu \alpha_1 x_1 + \left( \mu \alpha_1 + \alpha_2 \right) x_2 + \left( \mu + \alpha_1 - \frac{1}{T} \right) x_3 \end{bmatrix}
\]

Thus, we obtain the following overall control input.

\[
u_p = \frac{T}{L_{\delta_0}} \begin{bmatrix} \dot{x}_p + \dot{\dot{\theta}}_p + \mu \alpha_2 \dot{x}_1 \\ + \left( \mu \alpha_1 + \alpha_2 \right) x_2 + \left( \mu + \alpha_1 - \frac{1}{T} \right) x_3 \end{bmatrix}
\] (53)
Figure 4 shows the block diagram of the linear adaptive controller with respect to roll rate. It also indicates that the error equation of the system is linearized using the DAC observer and the satisfactory error response is acquired using the subspace stabilization technique.

In the same manner discussed above, the same block diagram of the linear adaptive controller for roll rate can also be obtained.

Fig.4 Block Diagram of Linear Adaptive Controller

7. NUMERICAL SIMULATION

To examine the effectiveness of the proposed system, a numerical simulation was performed with the ALFLEX simulation model. As a simulation model, a nonlinear flight dynamics of 6 degrees of freedom written in the Mat-lab/Simulink environment was used [9]. The initial conditions employed in this simulation were the same as those for the actual ALFLEX experiments performed in Australia.

Figures 5-6 shows the time histories of the error response of the roll rate $p$ and yaw rate $r$. To examine the effects of disturbance, we compared the responses without wind and with head wind or cross wind. ALFLEX follows the reference value regardless of disturbance using the proposed system. As the wind model, the MIL-F-9490D, MIL-F-8785C wind model was utilized.

Figure 7 shows the estimation results of the DAC observer. Reviewing this, it is observed that the observer can estimate the disturbance, uncertain and coupling terms satisfactorily although the estimated values vary from their true values.

Figures 8-9 show the vertical and horizontal loci of the center of gravity of ALFLEX. On longitudinal motion control, ALFLEX follows the reference path very well from approximately 2000[m] ahead of the tip of the runway and successfully touches down on the runway. On the other hand, on lateral motion control, the vehicle is affected by cross wind and largely deviates from the allowable range. The reason is that only roll rate and yaw rate are controlled and lateral position is not controlled in the proposed method at the present stage.

8. CONCLUSIONS

An adaptive flight control system employing the DAC observer and linear adaptive control methods was presented. The simulation results revealed that the proposed system is effective against disturbance and has a quick response of the controlled outputs, particularly, angular velocity in fast-time-scale state variables. Moreover, it was revealed that the DAC observer could estimate the disturbance, uncertain and coupling terms satisfactorily. In summary, the simulation showed that the proposed system could directly control nonlinear dynamics, possess robustness against disturbance, and follow a given reference trajectory quickly. In addition to the present method, if the MTS and STS subsystems are included in the system, satisfactory lateral position control is expected; however, this is a subject for future studies.

REFERENCES

Fig. 5 The time histories of the error response of the rolling rate.

Fig. 6 The time histories of the error response of the yawing rate.

Fig. 7 Behavior of the estimation roll rate of DAC observer.

Fig. 8 Horizontal flight path and reference path.

Fig. 9 Lateral trajectory, and upper and lower limits.